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Portfolio optimization using Mixture Design of Experiments: Scheduling trades within electricity markets

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ABSTRACT

Deregulation of the electricity sector has given rise to several approaches to defining optimal portfolios of energy contracts. Financial tools – requiring substantial adjustments – are usually used to determine risk and return. This article presents a novel approach to adjusting the *conditional value at risk* (*CVaR*) metric to the mix of contracts on the energy markets; the approach uses *Mixture Design of Experiments* (MDE). In this kind of experimental strategy, the design factors are treated as proportions in a mixture system considered quite adequate for treating portfolios in general. Instead of using traditional linear programming, the concept of *desirability function* is here used to combine the multi-response, nonlinear objective functions for mean with the variance of a specific portfolio obtained through MDE. The maximization of the *desirability function* is implied in the portfolio optimization, generating an efficient recruitment frontier. This approach offers three main contributions: it includes risk aversion in the optimization routine, it assesses interaction between contracts, and it lessens the computational effort required to solve the constrained nonlinear optimization problem. A case study based on the Brazilian energy market is used to illustrate the proposal. The numerical results verify the proposal's adequacy.

1. Introduction

In the last two decades, the electrical power systems of countries around the world have undergone many transformations. The impetus behind such transformations has been to introduce market mechanisms to a sector traditionally under government administration. Deregulating this sector has allowed the emergence of a market for electricity and with it a need for a good strategy of buying and selling electrical energy (Ramos-Real et al., 2009; Carpio and Pereira, 2007). In fact, this is deregulation's basic objective — to maximize the efficiency of electricity generation and transmission and thereby lower its costs (Oliveira et al., 2008).

The main problem for the electricity producers is hitting on the best strategy for selling electricity through a portfolio of contracts. Designing an optimal portfolio has been the focus of many papers. Among them can be cited the Markowitz mean-variance model, establishing the optimal strategy for minimizing the risk and maximizing the return (Badri et al., 2007). Two others include the variance–Skewness–Kurtosis-based portfolio optimization (Oliveira et al., 2007) and the use of genetic algorithm and multi-objective optimization (Lai et al., 2006).

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How to allocate different assets in a profitable portfolio is one of the major interesting issues in many areas including electricity market (Delarue et al., 2010; Galvani and Plourde, 2010; Polak et al., 2010; Green et al., 2010; Huisman et al., 2009; Liu and Wu, 2007).

Markowitz (1952, 1959) is known as the father of modern portfolio theory. He proved the fundamental theorem of mean-variance portfolio theory, which holds a constant variance and maximizes expected return (Delarue et al., 2010; Elton and Gruber, 1997). The objective was to develop a portfolio that maintained the expected return but with less risk.

In the electricity market the mean-variance portfolio (MVP) has been used extensively to manage portfolios of contracts that are based on the expected future consumption profile of a company or a pool of clients (Huisman et al., 2009). Möller et al. (2010) employed MVP to determine strategic positions in the balancing energy market and in identifying corresponding economic incentives in an analysis of the German balancing energy demand. In the energy market, MVP has also been employed to indicate that futures for crude oil, natural gas, and unleaded gasoline fail to enhance the performance of the representative energy stocks (in terms of return to risk), but do decrease the overall level of risk exposure borne by passive equity investors (Galvani and Plourde, 2010). Roques et al. (2008) applied the MVP combined with Monte Carlo simulations to identify, using their investment returns, optimal base load generation portfolios for large electricity producers in liberalized electricity markets. Awerbuch and Berger (2003) used the

MVP to contribute to the European Union Electricity Planning and Policy-Making, suggesting diversified generating optimal portfolios aiming to minimize society's energy price risk.

While Markowitz's seminal work (1952, 1959) has been used extensively, many authors have contributed to it and modified it in important ways. Based on the risk metrics defined by Artzner et al. (1999), one modification that fits very well for electricity is the *conditional value at risk* (*CVaR*), which was used by Badri et al. (2007). Another important feature of the portfolio theory is the assessment of the efficient frontier by means of quadratic programming or stochastic dual dynamic programming (Oliveira et al., 2007). These approaches, however, require high computational efforts and the results do not incorporate a confidence interval into the optimal value of a contract. In such cases, the producer has to contract exactly the value obtained from a mathematical model. When this value fluctuates, the maximum profit decreases.

This article uses Mixture Design of Experiments (MDE) to define an optimal contract portfolio for a given company. This technique has the advantage of considering a multivariate approach that returns a comprehensive index of desirability for the Total Present Value *TPVT* and for the *CVaR*. This index considers the amount of each asset in a portfolio and also the interactions between the assets.

This paper is organized as follows: Section 2 discusses the methodology used for generate optimal portfolios based on the combination of MDE and MVP, using conditional value at risk (*CVaR*) as a risk metrics; this section also presents an overview about the traditional portfolio optimization based on the mean-variance approach, the concepts of value at risk (VaR) and (*CVaR*), the model building strategy using Mixture Design of Experiments (MDE) and the desirability function; Section 3 describes the results obtained with the application of this methodology on Brazilian electrical sector; and Section 4 gives the conclusion of this paper.

2. Portfolio contracts optimization based on Mixture Design of Experiments

Portfolio theory seeks to manage risk in a group of assets to determine a combination that offers the lowest risk and the highest expected return. Such a group is called an optimal portfolio (Oliveira et al., 2008). A portfolio of assets is a combination of all potential assets each one with rate of return, r_i $(i=1,\ldots,n)$. The portfolio's return (denoted by r_c) is the weighted average of the component asset return with the investments proportions as weights (denoted by w_i), on this way, $r_c = \sum_i^n w_i r_i$. The mean of the probability distribution of the return, or the expected return, is an indication of the expected profitability. The variance of the distribution indicates how widespread the possible outcomes around the mean are — the larger the variance, the more uncertain the outcome. Therefore, the variance of the distribution can be used as an indication of the risk involved. The expected return of an asset and its variance can be written as $E(r_c) = \sum_{i=1}^n w_i E(r_i)$ and $\sigma^2(r_c) = \sum_{i=1}^n w_i^2 \sigma_i^2 + \sum_{j \neq i} \sum w_i w_j \sigma_{ij}$, respectively. The covariance σ_{ij} measures how many of the returns on two assets move in relation to each other. This is the well known Markowitz mean-variance model approach (MVP), establishing the optimal strategy for minimizing the risk and maximizing the return (Badri et al., 2007).

Suppose now the weights or amounts w_i of the MVP model are considered proportions of a mixture whose sum of weights is unitary or constrained to a specific bound, such as $\sum_{i=1}^n w_i = 1$ or $\sum_{i=1}^n w_i = \xi$. This is exactly the case of an experimental design called "Mixture Design of Experiments" (MDE).

MDE is a special type of response surface experiment in which the factors are proportions of components in a mixture (Myers and Montgomery, 2002; Cornell, 2002). These proportions are not negative, and if they are expressed as a fraction of the total mixture, the sum must be equal to one. The space formed by the mixing experiment for components described as a *simplex coordinate system*. The vertices of this convex region represent the pure mixture; the points inside the region are mixtures in which none of the components is missing. The centroid is the mixture with equal proportions of each component.

The mixture experiment allows establishing the relationship between the response variables and the relative proportion of components in terms of a mathematical equation which provides the identification of the influence of the proportion of each component and its combination with other components on the response variable. Generally, the functional relationship between the response variable and the proportions of q components is the defined by a polynomial of degree m, which can be a linear, quadratic, or cubic depending on the goals of practitioner. Eq. (1) shows a special cubic model.

$$E(x) = \sum_{i=1}^{q} \beta_i^* x_i + \sum \sum_{i<j}^{q} \beta_{ij}^* x_i x_j + \sum \sum_{i<j<k} \sum^{q} \beta_{ijk}^* x_i x_j x_k \tag{1}$$

The coefficients β_i^* show how each component contributes to the response variable. In the same fashion, term β_{ij}^* indicates what is the combined effect of components i and j. Indeed, for the linear model $\beta_{ij}^* = \beta_0 + \beta_i$ and for the quadratic model, it is possible to write: $\beta_i^* = \beta_0 + \beta_i + \beta_{ii}$ and $\beta_{ij}^* = \beta_{ij} - \beta_{ii} - \beta_{ij}$. These coefficients are estimated using the Ordinary Least Squares algorithm.

As mentioned earlier, there is a relationship between the (q, m) *simplex* and (q, m) *polynomial* so that there is a correlation between the number of points in the simplex and the number of terms in the polynomial. Thus, the parameters or coefficients of the polynomial can be determined by the values obtained for the response variable for each point in the simplex (Cornell, 2002).

When we compare the MDE to the traditional MVP approach, it is straightforward that the mean-variance equations may be written as a mixture response surface, where the amounts of capital investment in q contracts or assets are defined by the type of mixture design (extreme vertices, simplex lattice, or simplex centroid). However, this is not enough. Despite its gradual acceptance and dissemination, the mean-variance model has some drawbacks. First of all, MVP penalizes both positive and negative deviations from the average, the variance may not be best suited for measuring the risk of a portfolio. The return variability, when positive, should not be penalized; investors worry not about high returns but about low ones. The problems facing portfolio optimization arrive in two stages: the first, defining a metric of risk; the second, using this metric in the optimization model.

One risk metric that encompasses the potential loss of an investment is the *value at risk* (*VaR*). The *VaR* is the maximum amount of money that may be lost on a portfolio over a given period of time, with a given level of confidence (Marimoutou et al., 2009). This metric describes the loss that can occur over a given period at a given confidence level, due to exposure to market risk (Lai et al., 2006). As explained in 1), the first step in measuring the *VaR* is to select the time horizon and level of confidence (Rockafeller and Uryasev, 2001).

Although the *VaR* provides information about expected losses over a period of time for a given confidence level, the losses that exceed the *VaR* are unknown. Moreover, the *VaR* metric is not a consistent measure of risk, which means it is not sub added. Finally, the *VaR* is not a convex function of hard minimization (Artzner et al., 1999; Dalhgren et al., 2003). Nevertheless, an adjustment may be made to the *VaR*, changing it to another metric known as *conditional value at risk*, *CVaR* (Rockafeller and Uryasev, 2001; Dalhgren et al., 2003) defined by Eq. (2):

$$\phi_c(w) = (1-c)^{-1} \int_{f(w,y) \geq v_c(w)} f(w,y)p(y)dy \tag{2}$$

where c is the confidence level; $\upsilon_c(w)$ is the *VaR*; $f(w,y)$ is the return function; $p(y)$ is the probability density induced by the uncertainties of the y variable (in this case the spot price).

In Eq. (2), the probability of $f(w,y) \geq \upsilon_c(w)$ is equal to $(1-c)$. Thus, $\Phi_c(w)$ comes out as the conditional expectation of the loss associated with w, relative to the loss being $\upsilon_c(w)$ or greater. In this way, we may characterize $\Phi_c(w)$ and $\upsilon_c(w)$ in terms of the function F_c which may be integrated by sampling the probability distribution of y according to its probability density $p(y)$. If the sampling creates a collection of vectors y_k, where $k=1$ to q, the approximation of $F_c(w,\upsilon)$ results in Eq. (3).

$$F_c(w,\upsilon) = \upsilon + \frac{1}{q(1-c)}\sum_{k=1}^{q}\left[f(w,y_k)-\upsilon\right] \tag{3}$$

The optimization problem written in terms of *CVaR* can be solved by linear programming techniques with large numbers of both assets and scenarios. Employing such techniques requires complex methods: (*cutting plans*) (Mansini et al., 2001; Pereira, 2002), *benders decompositions* (Birge and Loveaux, 1997; Ruszczynski, 1997) and still others (Sen and Higle, 1999). Moreover, applying these takes enormous computational effort while the optimal point obtained may not even be feasible for the company. For example, if the company is an agent of generation, the response variable w_i, which represents how much must be sold through bilateral contract i, can be a value that is difficult to negotiate for the company. In others words, the mathematical response is difficult to implement.

Then, is this novel approach, the portfolio variance σ_i^2 can be written in terms of *CVaR* with weights or amounts w_i defined by the chosen MDE.

The design of an optimal portfolio is also a nonlinear multi-objective optimization task treating the simultaneous minimization of the risk and maximization of the return. To accomplish with this objective, risk and return equations are generally combined in a form of a utility function, such as $U=E(r_c)-0.5\lambda\sigma^2(r_c)$, where λ is the weighting factor that reflects the decision maker's preference or risk aversion (Roques et al., 2008). When dealing with MDE, we can also adapt the concepts of return, risk (*CVaR*) and utility for the energy market through *desirability function*, a multi-objective optimization transformation which allows the combination of risk and return and also allows the introducing of the investor's aversion to risk (λ). Additionally, one can define regions and ranges of responses for each variable where the maximum return and minimum risk do not vary.

Available in several statistical packages, desirability function is a transformation of each estimated response variable of interest $\left(\hat{Y}_l\right)$ to a desirable individual value, (d_i), which varies from 0 to 1 according to the closeness of the solution to the established targets. This transformation is obtained in terms of lower (L_i) and upper bounds (H_i) or target (T_i) chosen for each response. After that, the individual transformations are combined through a geometric mean with the degree of importance of each responses (ζ_i), chosen according to the practitioner's interest. In fact, ζ_i indicates the importance of each property in relation to the others in the multi-objective optimization. The D value measures the overall desirability and must lie in the interval [0, 1] (Derringer and Suich, 1980; Myers and Montgomery, 2002).

Considering the Markowitz mean-variance approach using *CVaR* as a risk metrics and adopting the transformations obtained through desirability functions, a portfolio contracts optimization approach based on Mixture Design of Experiments could be written as:

$$\begin{aligned} &\textit{Maximize} \quad D = \sqrt{d_{TPVT} \times d_{CVaR}} \\ &s.t.: \quad d^{n+1}(y_i) \geq D, \quad i = 1,2,\ldots,k \\ &\qquad D \geq 0 \\ &\qquad x \in \Omega \end{aligned} \tag{4}$$

With:

$$d_{TPVT} = \begin{cases} 0 & \text{if} \quad TPVT_i < L_i \\ \left[\dfrac{(TPVT_i - L_i)}{(T_i - L_i)}\right]^{\lambda} & \text{if} \quad L_i \leq TPVT_i \leq T_i \\ 1 & \text{if} \quad TPVT_i > T_i \end{cases} \tag{5}$$

$$d_{CVaR} = \begin{cases} 0 & \text{if} \quad CVaR_i > H_i \\ \left[\dfrac{(H_i - CVaR_i)}{(H_i - T_i)}\right]^{\lambda} & \text{if} \quad L_i \leq CVaR_i \leq T_i \\ 1 & \text{if} \quad CVaR_i < T_i \end{cases} \tag{6}$$

$$(TVPT)_{i,j} = \sum_{t=1}^{T}\sum_{i=1}^{I}\frac{1}{(1+tax)^{t-1}} \times \left\{ \sum_{\substack{n=1 \\ h_n=1 \\ k_n \geq t \\ J_{n\leq t}}}^{N} \left(P_i - \pi_t^{i,s}\right)x_n + \left[\left(\pi_t^{i,s} - COP\right) \times D_t^{i,s}\right] \right\} \tag{7}$$

$$(CVaR)_{i,j} = F_c(w,v) = v + \frac{1}{q(1-c)}\sum_{k=1}^{q}\left[f(w,y_k)-v\right] \tag{8}$$

$$(CVaR, TPVT)_{\text{portfolio}} = \sum_{i=1}^{q}\beta_i^* x_i + \sum\sum_{i<j}^{q}\beta_{ij}^* x_i x_j + \sum\sum_{i<j<k}\sum^{q}\beta_{ijk}^* x_i x_j x_k \tag{9}$$

Where:

$d^{n+1}(y_i)$	is the desirability function of the y_i on $(n+1)$th run;
Ω	denotes the lower and upper bounds chosen for the proportions of each contract.
L_i	desirability lower bound.
T_i	desirability target.
H_i	desirability upper bound.
λ	desirability weights (risk aversion).
$D_{t,s}$	is the dispatch of the company at time t for the s series.
x_n	is the energy volume that may be traded through n bilateral contract candidates.
$P=(P_1,\ldots, P_n)$	is the energy price associated with the n bilateral contract candidates.
COP	represents the plant generation costs.
J_n	is the start time of contract candidate n.
K_n	is the contract's end time.
i	is the ith energy submarket.
tax	is the income tax or the cost of capital (%).

In Eq. (7), TVPT is calculated by generating random variables for the companies' cash flow, using the spot price for each time period $t=1,\ldots T$, for a set of spot price series $s=1,\ldots, S$ in a specific submarket $i=1,\ldots, I$.

In the context of a desirability function, λ is a weight that emphasizes the interest of the practitioner in optimization, indicating his preference by the target value or the bounds. For the minimization, less emphasis is placed on the target when the weight is less than one. Acceptable is any solution found by the algorithm between the target and the upper bound. A weight equal to one places the same emphasis on the target and the bounds. Otherwise, a weight greater than one forces the algorithm to find a solution as near as possible to the target. For maximization problems, the approach is similar.

Then in the minimization of *CVaR*, λ can be considered the coefficient of risk aversion. Hence, the larger that λ can be fixed for d_{CVaR}, the more conservative will be the investor profile, since the desirability will only equal one if the target (a minimum desirable value) is achieved. λ less than one indicates little risk aversion, and any solution found between the target and the lower bound will be reasonable and acceptable. This implies a large *CVaR* value.

The main advantage of treating λ as a risk aversion coefficient is the fact that an investor can easily simulate several scenarios for the portfolios using available statistical software packages like Minitab.

From the aforementioned discussion, the following framework may be applied by the practitioner when dealing with Energy Portfolio Contracts:

- Choose the numbers of contracts to build the portfolio and the mixture design.
- Choose a specific mixture design (extreme vertices, simplex lattice, or simplex centroid) according to the number of components and defining if the adopted proportions must be constrained or not.
- Calculate the values of $TVPT_i$ and $CVaR_i$ for each experiment generated by MDE according to the Eqs. (6) and (7).
- Fit adequate models for *TVPT* and *CVaR* analyzing statistically the *mixture models* and searching for the higher R^2_{adj}.
- Obtain the predicted value $\hat{Y}_i$ of the *i*th response of interest (*VPTP* and *CVaR*).
- Choose the desired values for T_i, H_i and L_i.
- Apply the desirability function of maximization for *TPVT* to obtain d_{TPVT} according to Eq. (5).
- Use the desirability function of minimization for *CVaR* to obtain d_{CVaR} using Eq. (6).
- Establish a value for the risk aversion, such as $0.1 \le \lambda \le 10$
- Run the nonlinear optimization routine based on the overall desirability (*D*) to obtain the feasible solution.
- If desirable, simulate several values of λ to the portfolio contracts.

As will be seen in the numerical example, the application of Mixture Design of Experiments considers several scenarios and predicts how each contract will improve the generator gain.

3. Mixture Design of Experiments applied to portfolio of contracts in Brazilian electrical sector

In Brazil, restructuring the power system introduced new agents: the National Regulatory Agency (ANEEL), the Independent System Operator (ONS), the Wholesale Electricity Market (CCEE) and the Energy Planning Institution (EPE). More recently (2004), Law 10.848 was enacted establishing new rules for the Brazilian wholesale market. One major change introduced two trade environments: the Regulated Trade Environment (RTE) and the Free Trade Environment (FTE). The RTE was designed for the captive consumers represented by the distribution companies. The agencies ANEEL and CCEE, acting on the behalf of the distribution companies, conduct centralized auctions for buying electricity.

The prices at the FTE are called market clearing prices (MCP) and are set by the marginal cost of the energy. This cost is derived from an optimization program called NEWAVE (Paravan et al., 2004; Joy et al., 2004). Although the market sets the prices of bilateral contracts at the FTE and for auction bids at the RTE, the MCP serves as a guide for contracts in both environments.

The MCPs, i.e., the spot prices, are sensitive to the water inflows at the reservoirs of hydropower plants. Such sensitivity makes them volatile. 67.53% of Brazil's produced electricity comes from hydro-generation. The inflow uncertainties and the consequent volatility in electricity prices create a need for market players to hedging bets. By using bilateral contracts, for instance, the generators can sell their energy to the distributors under RTE or directly to a free end-consumer. They can thus avoid exposure to the volatility of MCP under FTE.

Brazil's market system is known as “tight pool.” This means that the National System Operator (NSO) determines the dispatching of plants as well as the spot price $\pi_{t,s}$, using *Newave* Software. In fact, *Newave* is used to plan the operation of the electrical system and implements dual stochastic dynamic programming to minimize the total cost of the system's operation (Carpio and Pereira, 2007). It generally uses five years as a time horizon.

The restructuring process in Brazil started in 1997 when the wholesale market was established. In 2004, after the rationing that took place in 2001, the government changed the market rules trying to basically ensure long-term investment (Ramos-Real et al., 2009; Carpio and Pereira, 2007). The elements of this new regulatory policy seek to promote joint use of available resources for energy generation (including renewable), strategies in relation to demand/contract to ensure a level of security and, finally, the integration of agents of monitoring and an evaluation system for the short and long-term. The current structure, according to Fig. 1, covers two areas of engagement: the Regulated Trade Environment (RTE) and the Free Trade Environment (FTE).

The FTE maintains the rules of the previous wholesale market. The new RTE is managed through auctions by the government, resembling the single buyer model. At the end of the auction, negotiations to purchase and sell energy are formalized by the CCEE through bilateral contracts. These contracts are known as Contract for Sale of Energy in a Regulated Environment (CSERE). The environment of free contracting, negotiations, and bilateral contracts is freely established between free customers and producers. Thus, producers can sell energy on the FTE or the RTE.

In addition to their long-term contracts, agents can sell electricity on the spot market, which is included in the FTE. The rules were designed, however, to let agents avoid exposure to the spot market's volatility; e.g., distributors are obliged to enroll in long-term contracts of 100% of their forecasted load. For the spot market, one of the most important variables is the Market Clearing Prices (MCP). The MCP, due to the predominance of hydro-generation, is quite volatile; its level is connected with the amount of rainfall. In periods of low inflow at the reservoirs, the MCP is quite high. The ANEEL provides a ceiling and floor for the MCP. In 2008 the ceiling was $291.36/MWh and the floor was $7.66/MWh.

Based on these characteristics of the Brazilian market, electricity producers initially have three alternatives: 1) selling their energy to the distributors participating in the government auctions, 2) selling directly to the free consumers, or 3) waiting to sell on the spot market. This situation is analogous to an investor having three types of assets to invest, with all the risk that attends each. The optimal portfolio determines a point of operation that considers the total amount of energy generated by the plant, a plot for the bilateral contracts derived from the auctions or agreed to directly with the free consumers, and the remainder energy going on the spot market.

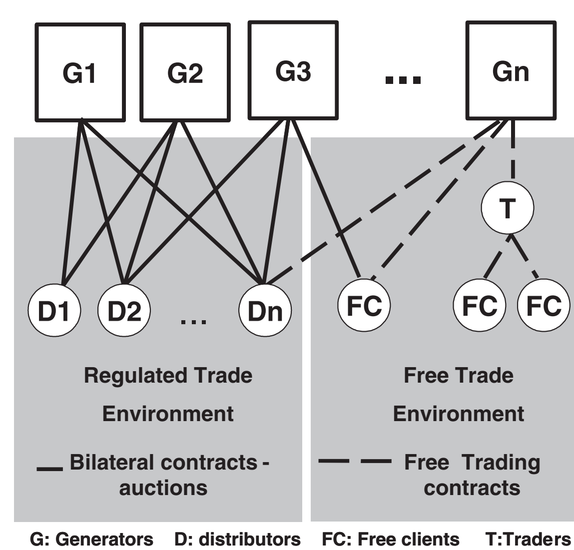

Fig. 1. Brazilian electrical sector.

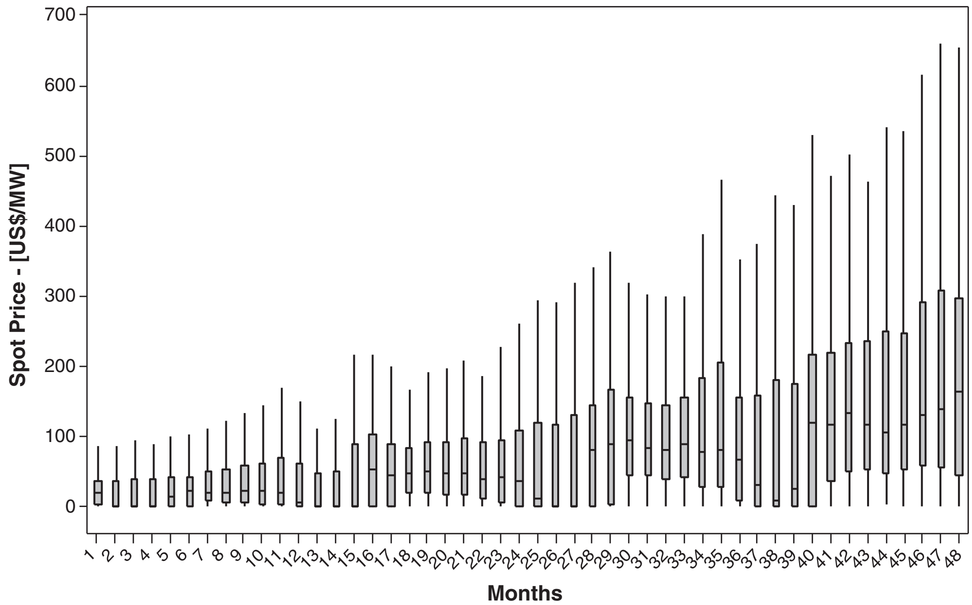

Fig. 2. Spot price from Jan 2007 to Jan 2011.

The theory of portfolio optimization can help producer's decision-making process regarding minimizing risks and maximizing returns. This theory must be adapted to the electricity sector. Specifically in Brazil the MCP is not freely obtained from the dynamic between supply and demand, but from a computational model. Moreover, there are rules that also affect the market. One example is the case of a producer with operating costs higher than the MCP. In such a situation, the energy is not dispatched. This agent will have to buy on the spot market the total energy that the company traded through bilateral contracts.

The following input variables are considered on the *Newave*: initial levels of the reservoir, demand for energy by the subsystem, cost of thermal classes of fuel, setup of hydraulic power plants, and setup of thermal power plants, the possible exchanging of energy between the subsystems, and, finally, the expansion or increase in supply (new power plants built). Based on the report *Programming Monthly Operation* (*PMO*), under ONS, the input variables are released monthly. The simulation using *Newave* used the data provided by the January 2007 report. Fig. 2 shows the evolution of the spot price for the considered time.

The spot price presents a trend and greater dispersion for the months at the end of the planning period. This volatility in prices makes the generator revenue fluctuate as well.

Table 1 presents the characteristics of the bilateral candidate contracts considered in this study. In this table, one can see the demand in MWh and as a percentage of the capacity of the plant. All the contracts were performed in the southeastern and central western region (SE/CO). The starting month was January (01).

Table 1
Candidate contracts to compose portfolio.

Contract	End Month	Price $/MWh	Maximum volume MWh	Maximum volume − % *U*
I_1	12	42.50	150	0.300
I_2	24	41.25	457	0.914
I_3	48	49.10	294	0.588
I_4	12	42.50	239	0.478
I_5	24	38.30	74	0.148
I_6	48	39.10	47	0.094

The company has a thermal gas power plant with an installed capacity of 500 MW. The plant is considered flexible without any minimum dispatch. The plant's operating cost is the sum of fuel costs and the cost of operation and maintenance, equaling $32.50/MWh. The study's horizon is forty-eight months. The company can trade in the spot market, I_s, considering the spot price. As we have a higher constraint for each component, representing the maximum demand in each contract, we should use the experiment called *Mixture Extreme Vertices Design* (Wheeler and Chambers, 1992; Piepel and Cornell, 1994; Wheeler, 1995; Chongqi, 2001; Sadeghi and Shavalpour, 2006).

Table 2
TPVT and *CVaR* coefficients.

Contracts	Coefficients β^*_{ij} *TPVT*	*CVaR*
I_1	265.83	290.71
I_2	216.21	261.48
I_3	234.33	230.33
I_4	265.83	225.15
I_5	161.96	−22.02
I_6	156.42	1631.36
I_s	203.88	−2.07
$I_1 \times I_2$	–	−0.82
$I_1 \times I_3$	–	−1.11
$I_1 \times I_4$	–	−0.39
$I_1 \times I_5$	–	0.42
$I_1 \times I_6$	–	−4.14
$I_1 \times I_s$	–	−0.80
$I_2 \times I_3$	–	0.28
$I_2 \times I_4$	–	−0.83
$I_2 \times I_5$	–	1.20
$I_2 \times I_6$	–	−2.81
$I_2 \times Is$	–	0.07
$I_3 \times I_4$	–	−0.92
$I_3 \times I_5$	–	1.23
$I_3 \times I_6$	–	−2.61
$I_3 \times Is$	–	−0.34
$I_4 \times I_5$	–	0.38
$I_4 \times I_6$		−4.20
$I_4 \times I_s$		−1.37
$I_5 \times I_6$		−2.99
$I_5 \times I_s$		1.59
$I_6 \times I_s$		−7.20

Table 3
Statistical properties of *TPVT* and *CVaR* models.

	S	R^2 (%)	R^2_{pred} (%)	R^2_{Adj} (%)
TPVT	0.0108	100	100	100
CVaR	8359	98.6	97.3	98.3

Thus, according to the number of components in the portfolio (Table 1) and considering the trading on the spot market (I_s), the experimental region has 66 vertices 176 edges and 256 faces. The experimental data has 141 runs and statistical analysis is performed using *Minitab*® 14. Table 2 presents the model coefficients for the invested amounts in each candidate contract and the responses *TPVT* and *CVaR* with $I=1,2,...,7$ and $j=1,2,...,7$ and $I_s=I_7$.

Actually, in the experimental design, the analysis of variance was used and the hypotheses were tested to check the influence of an investment in a particular contract and of the variables *TPVT* and *CVaR*. As the significance level is 0.05, all factors and interactions among the factors that have a *P-value* less than 0.05 cause a significant effect on the variables. The equations that describe the behavior of *TPVT* and *CVaR* are given below.

$$TPVT = 265.83I_1 + 216.21I_2 + 234.33I_3 + 265.83I_4 + 194.65I_5 + 156.42I_6 + 203.88I_S \quad (10)$$

$$\begin{aligned} CVaR = {} & 290.71(I_1) + 261.48\ I_2 + 230.33\ I_3 + 225.15\ I_4 \\ & + 225.02\ I_5 + 1631.36\ I_6 - -2.07 I_s - 0.82\ I_1 \times I_2 \\ & + 0.42\ I_1 \times I_5 - 4.14\ I_1 \times I_6 - 0.80\ I_1 \times I_5 + 1.2\ I_2 \times I_5 \\ & + -2.81 I_2 \times I_6 + 0.07\ I_2 \times I_5 + 1.23\ I_3 \times I_5 - 2.61\ I_3 \times I_6 \\ & + 0.38\ I_4 \times I_5 - 4.21\ I_4 \times I_6 + -2.99\ I_5 \times I_6 + 1.59\ I_5 \\ & \times I_S - 3.60\ I_6 \times I_s \end{aligned} \quad (11)$$

The equations of *TPVT* and *CVaR* were obtained using an OLS algorithm. Table 3 presents the statistical properties of their models.

The statistical models for *TVPT* and *CVaR* present a reasonable power of explanation, revealing small values for the standard error *S* and large values for R^2 and R^2_{Adj}. Figs. 3 and 4 show that the residuals are *i.i.d.* (independently and identically distributed), normally distributed, meaning the models are reliable.

The optimal portfolio definition consists of determining which contracts should be chosen. Figs. 5 and 6 show the sensitivity analysis for the variables *TPVT* and *CVaR* using the *Cox response trace plot*.

Each component in the mixture has a corresponding trace direction. The points along one trace direction of a component are connected, thereby producing as many curves as the number of components in the mixture.

Response trace plots are especially useful when there are more than three components in the mixture and the complete response surface cannot be visualized on a contour or surface plot. The response trace plot can be used to identify the most influential component and it can be employed as a tool for a sensitivity analysis. The response trace plot shows how the responses change when the proportion of each component in the mixture increases.

In Fig. 5, the vertical axis represents the values for the return (*TPVT*) and the horizontal axis represents the change in the quantities of each candidate in the contract portfolio. The reference portfolio is represented as the zero point on the horizontal axis. According to this figure, contracts I_1 and I_4 are the most important to explain the return (*TPVT*). In fact, when increasing the amount invested in these contracts, the return of the portfolio also increases. On the other hand, the return of the portfolio falls when the amount invested in these contracts decreases. The investment on contract I_2 presents an opposite behavior. This occurs because I_2 has the lowest return per MWh, or \$12.3($\beta_2=\beta_2^*-\beta_0;\beta_0=203.88$).

A similar behavior occurs with I_5 and I_6. It can be seen, however, that the lines present a more pronounced slope. Thus small variations in the amount invested in these contracts greatly impact the *TPVT* because I_5 and I_6 have a negative average return per MWh, \$41.92 and \$101.08, respectively. For trading on the spot market, I_S, we may conclude that when the amount sold in the spot market decreases, the *TPVT* tends to increase. If participation in the spot market increases, the *TPVT* tends to decrease.

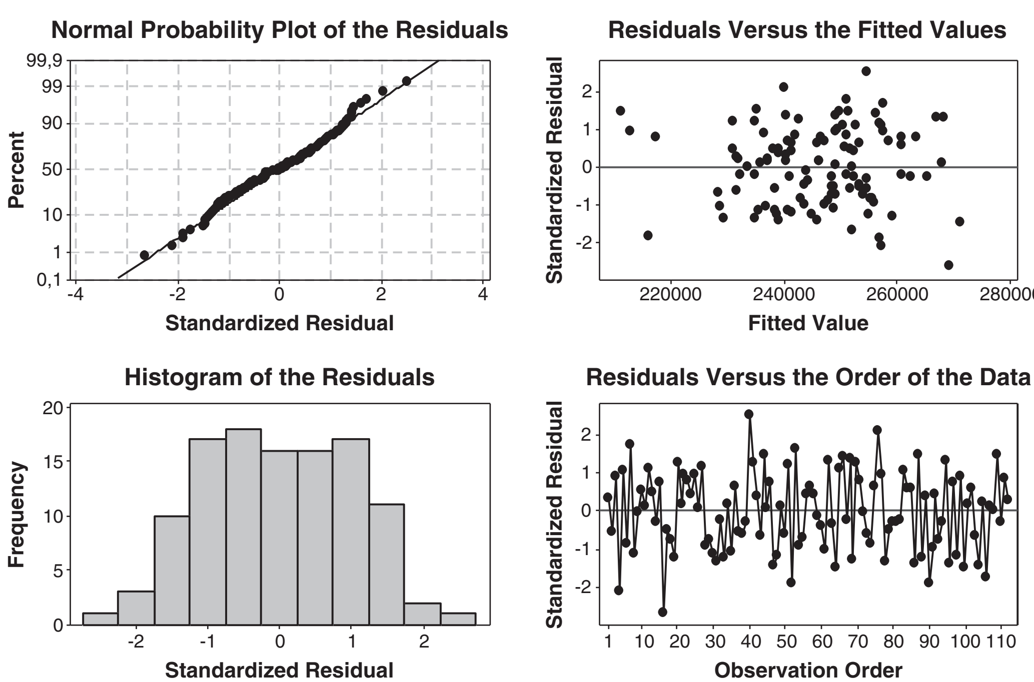

Fig. 3. Residuals plot for *TPVT*.

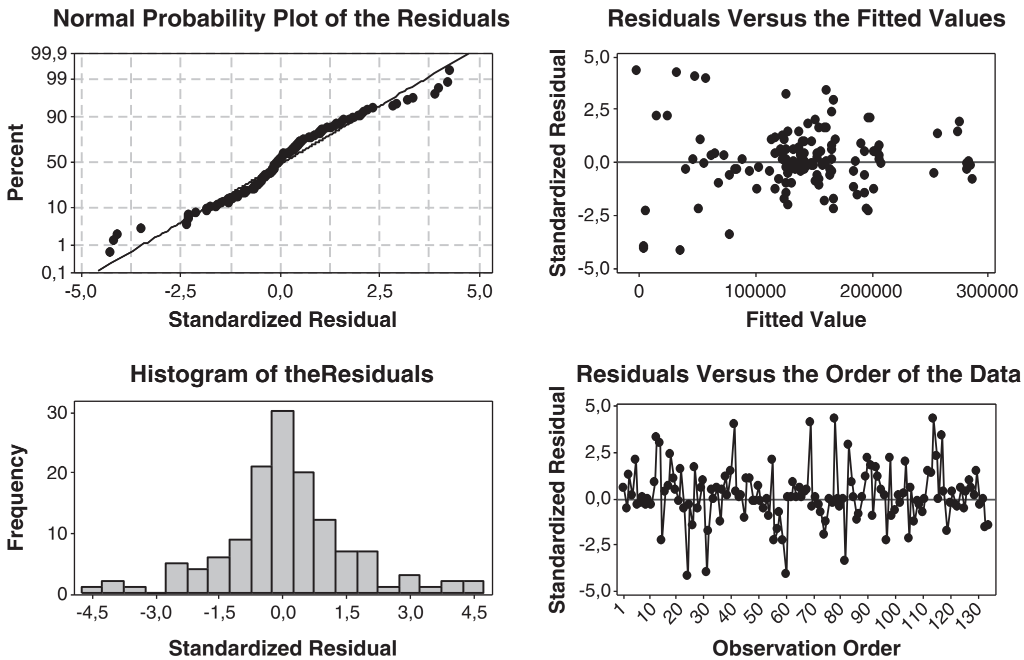

Fig. 4. Residuals plot for *CVaR*.

Considering *CVaR*, the contract I_3 is responsible for most of the risk. For trading contract candidates I_1 and I_4, the risk tends to decrease. For the contracts, I_2, I_5, and I_6, the variation in the amount invested always causes an increase in risk. Moreover, the contract I_2 contributes mostly to the increase in risk. Finally, trading on the spot market is an option that causes the largest drop in *CVaR*. Indeed, an alternative that enables greater *TPVT* and lower *CVaR* is to sell a certain amount of energy to the market and what's left to bilateral contracts, or diversifying the means to trade the energy generated by plants. Trading alternatives can be obtained using multi-objective optimization with the desirability function.

For *TPVT*, for example, one can set a lower bound of $112.676 with $115.493 for the target. For *CVaR*, the target can be defined as $56.338 and the upper bound as $61.033, since the objective function here must be minimized.

The desirability functions of *TPVT* and *CVaR* are given by Eqs. (5) and (6), respectively. The weights for the desirability function of *CVaR* and *TPVT* are equal to 10. This provides an important characteristic: it facilitates including the investor's aversion to risk. Thus, from the early development of the problem, the investor can feed the system real information, not approximated or idealized information. This gives flexibility to the portfolio optimization. In terms of desirability function, the transformations can be written as:

$$d_{TPVT} = \begin{cases} 0 & \text{if} \quad TPVT_i < 112676 \\ \left[\dfrac{(TPVT_i - 112676)}{(2817)}\right]^{10} & \text{if} \quad 112676 \le TPVT_i \le 115493 \\ 1 & \text{if} \quad TPVT_i > 115493 \end{cases} \qquad (12)$$

$$d_{CVaR} = \begin{cases} 0 & \text{if} \quad CVaR_i > 61033 \\ \left[\dfrac{(61033 - CVaR_i)}{(4695)}\right]^{10} & \text{if} \quad 56338 \le CVaR_i \le 61033 \\ 1 & \text{if} \quad CVaR_i < 56338 \end{cases} \qquad (13)$$

With the support of a computational tool, it is possible to solve this optimization problem, finding (in MWh): $I_1 = 80$; $I_2 = 0$; $I_3 = 141$, $I_4 = 112$; $I_5 = 38$; $I_6 = 24$ and $I_s = 104$, with $TPVT = \$116,620.00$ and $CVaR = \$37,244.13$ and $D = 1$. Besides the values of return and risk, it is possible to verify the stability of the optimum and also define optimal intervals for each quantity sold in each contract (in MWh): $0 \le I_1 \le 150$; $0 \le I_2 \le 230$; $0 \le I_3 \le 219$; $76 \le I_4 \le 239$; $0 \le I_5 \le 11$;

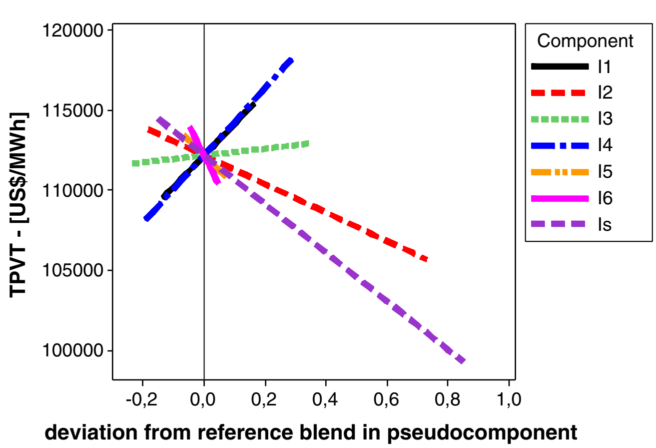

Fig. 5. Sensitivity for *TPVT*.

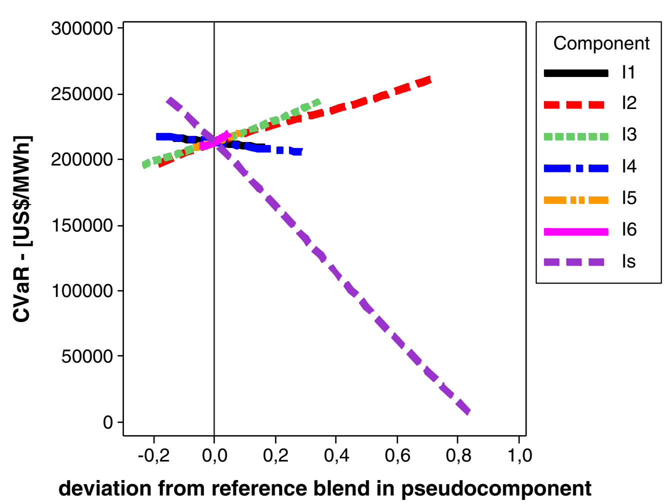

Fig. 6. Sensitivity for *CVaR*.

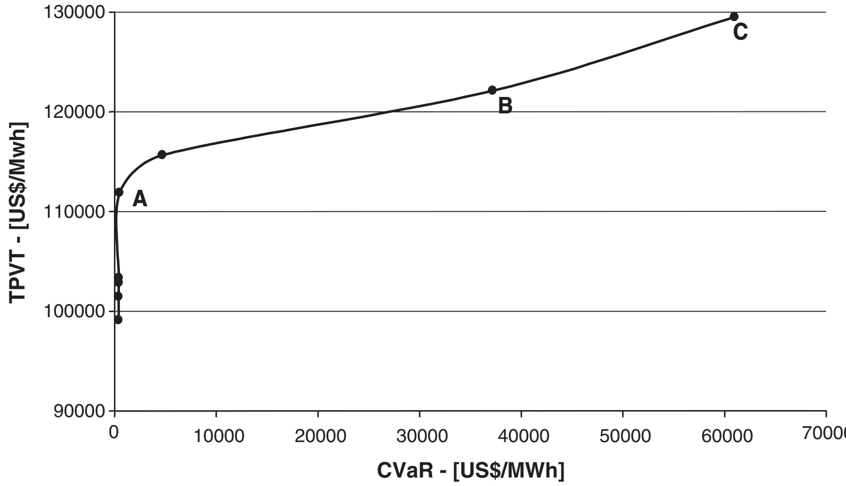

Fig. 7. Efficient frontier.

$0 \leq I_6 \leq 37$ and $42 \leq I_s \leq 172$. Fig. 7 describes the efficient frontier for the presented example.

To build up an efficient frontier we chose all solutions whose total desirability (D) equaled 1. This ensured the obtained optimum also corresponded to the maximum value of desirability. By selecting Points A, B, and C of the efficient frontier, it is possible to check how much energy is traded in each contract (Fig. 8).

Fig. 9 shows that for Points A, B, and C of the efficient frontier, the plant sells all of its capacity. For the remaining months, according to the level of return and risk requirements, the plant tends to decrease the amount of energy sold. This feature can be explained as follows: the contracts I_1 and I_3, with higher returns per MWh are established for a period of twelve months while the remaining ones are established for longer periods. Furthermore, trading in the energy market is an alternative only used to reduce risk in the portfolio, reducing the amount of energy sold in this kind of negotiation.

The demand profile of the plant suggests that the plant can schedule a future contract, starting on Day 13. The same occurs on Day 24. Thus, the company may sell in any period all of its production capacity. This analysis with future contracts, however, should be performed considering the investor's objective, that is, maximizing return and minimizing risk.

4. Conclusions

This work proposes a methodology for a producer trading energy on the electricity market. The methodology, used to optimize energy contract portfolios, uses *CVaR* as a risk measure. Mixture Design of Experiments (MDE) was used as a strategy to build up nonlinear models of risk and return according to the proportions of desired contracts. This approach is innovative when compared to traditional methodologies like linear or dynamic programming to obtain the efficient frontier for *CVaR*. The energy contract portfolio was modeled as a mixture problem, considering each contract as a component to explain the behavior of risk and return.

The first step of the methodology is to choose the number of contracts that the portfolio will comprise and also to choose the mixture design and the specific mixture design. The second step is to calculate the values of *TVPT* and *CVaR*, using them as return and risk equations. By combining these equations with contract proportions, we can obtain two response variables for MDE. These can be fitted by a statistical package using the Ordinary Least Squares (OLS) algorithm. Since the statistical models with a higher R^2_{Adj} are obtained, we must choose the desired values for the portfolio bounds T_i, H_i, and L_i. Applying the desirability function of maximization for *VPTP* and minimization of *CVaR*, and establishing a value to aversion risk coefficient λ, a nonlinear optimization routine can be used to obtain a feasible solution to the portfolio contracts.

This paper presented an example, with seven contracts candidates, from the Brazilian electricity market. In spite of the hypothetical application, it was possible to define the portfolio efficiency frontier for desirability equal to 1. In this way, the company's strategy of trading was defined as a period of 48 months ahead, taking into account the amount of bilateral contracts on the spot market.

Compared to similar methodologies, the present approach has the advantage of small computational effort and simple analysis of responses.

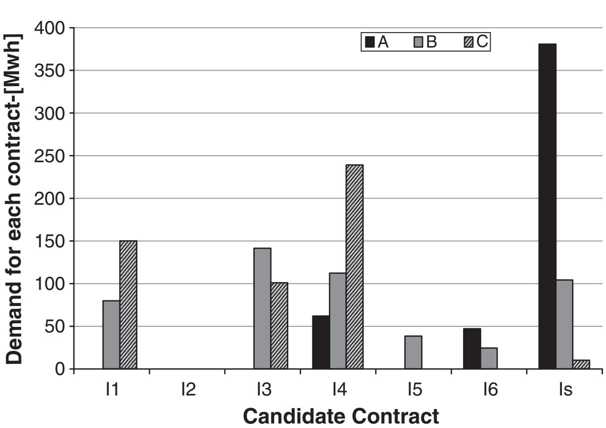

Fig. 8. Contracts mix variation.

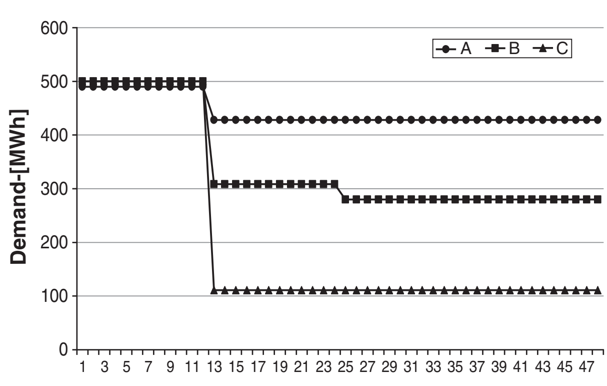

Fig. 9. Trading scheduling.

We here point out further advantages:

- Facilitates the inclusion of risk aversion in the form of coefficient, helping with the simulation of several scenarios for the portfolios using commercial statistical software's packages.
- Possibility of measuring the effects over *TPVT* and *CVaR* obtained with the Interactions between contracts.
- Possibility of embodying the problem with real information about the investment from the project's beginning.
- User-friendly, the desirability makes the decision-making problem easier to solve.
- The nonlinear equation system formed with desirability equations is easier to implement and less time consuming.
- Possibility of including processes or outer variables that can be in the study.

For future research, we suggest including process variables like options, collar and swaps in the Mixture Design of Experiments.

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