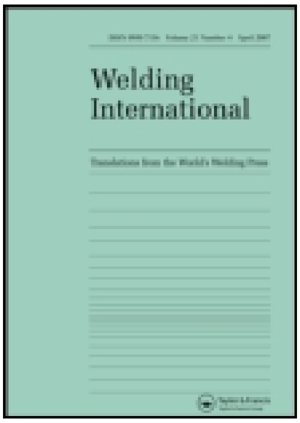


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# FCAW process optimization using the multivariate mean square error†

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Optimization of welding processes is not a trivial task, mainly due to the great number of required and desirable characteristics that must be analysed. Moreover, the optimization of a welding process with multiple characteristics without considering the variance–covariance structure may lead to inadequate optimum. To help with this task, a method of multi-objective optimization based on the multivariate mean square error applied in the study of multiple correlated characteristics of a flux-cored arc welding process is presented. This method characterized by a combined approach based on the response surface methodology, design of experiments, and principal components analysis consisted of an attempt to achieve the nearest values to specific targets, for each characteristic (penetration, deposition rate, deposition efficiency, convexity index of the weld bead, and dilution), considering the welding variables expressed as a result of welding voltage ($V$), wire feed speed ($Va$), and contact tip to workpiece distance ($d$). The results point, to a good adequacy of the proposed method.

**Keywords:** multivariate mean square error; FCAW; design of experiments; response surface methodology



## 1. Introduction

In a welding process, it is desirable to specify parameters appropriately in order to carry this out with the maximum degree of accuracy possible. The search for these parameters, however, involves an optimization process of multiple variables, representing multiple quality characteristics, either required or desirable, for a given process or product, comprising a task which is difficult to implement and of improbable efficacy. According to Wu[1] and Khuri and Cornell[2], the presence of possible relations of dependency (or correlation) between these various output characteristics of the processes may interfere with the specification of these parameters and conduct the process to inappropriate optimums resulting in equivocal and meaningless conclusions. According to Khuri and Conlon[3] and Bratchell[4], this inappropriateness is associated with the insufficiency of the method of ordinary square minimums at estimating the coefficients of multiple answers correlated simultaneously as a result of the strong influence of correlation structures on the transfer functions used.

In this respect, this paper presents the application of an approach employed by Paiva[5] based on a multivariate extrapolation of the method proposed by Köksoy and Yalcinoz[6] and Lin and Tu[7] for use of the mean square error (MSE) method in the simultaneous optimization of mean and variance, with the aim of determining ideal parameters for manufacturing processes. With the adaptations proposed, the so-called multivariate MSE (MMSE) is capable of constructing a mathematical model with an adequate set of optimum parameters, generated from the correlation structures existing between the responses, identifying these structures before this model is constructed.

The MMSE method proposed by Paiva[5] presents a framework for the application of a range of techniques and methodologies attempting to minimize efforts in the search for appropriate parameters for manufacturing processes.

According to Paiva[5], the multivariate optimization of these parameters includes the combined application of different methodologies such as planning and analysis of experiments (design of experiments (DOE)), the response surface methodology (RSM), and principal components analysis (PCA) in the problems of the nominal-the-best (NTB) type. In this case, DOE is used to study the behaviour of variables; RSM to model the approximation functions of optimum points, generally found in curvature areas according to their convexity and PCA in the construction of the MMSE index, with the aim of minimizing distances between responses and their respective goals and variances.

With the aim of discussing the efficiency of the application of this methodology and taking into account that such characteristics are typical of welding processes, a case of multi-objective optimization of multiple characteristics correlated with the flux cored arc welding (FCAW) process is investigated. It is worth pointing out that the results obtained did not aim to show the technological advance of the process but the possibility of the method being applied in other similar manufacturing processes. Both the methodology proposed and the case investigated are described in the following sections of this paper.

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## 2. Combined methodologies

Carrying out the optimization of multiple characteristics, taking into account a correlation structure if this exists, includes the application of combined methodologies which according to the model proposed by Paiva[5] must be employed to develop the experiments, find a square model representative of the problem, group together the multiple functions of the goal, and detect the correlation structure. After noting the existence of a correlation, one must then proceed with an analysis of the principal components (PCs) which play an important role here as this reveals correlation structures, even latent ones; find the square models of the PCs; specify specification limits in terms of the PCs; and finally apply the MMSE.

Figure 1 demonstrates the flow of procedures in the application of the various combined statistical and mathematical methodologies and the obtainment of ideal parameters for the MMSE model.

Following this, the methodologies involved in this approach of multivariate optimization are presented.

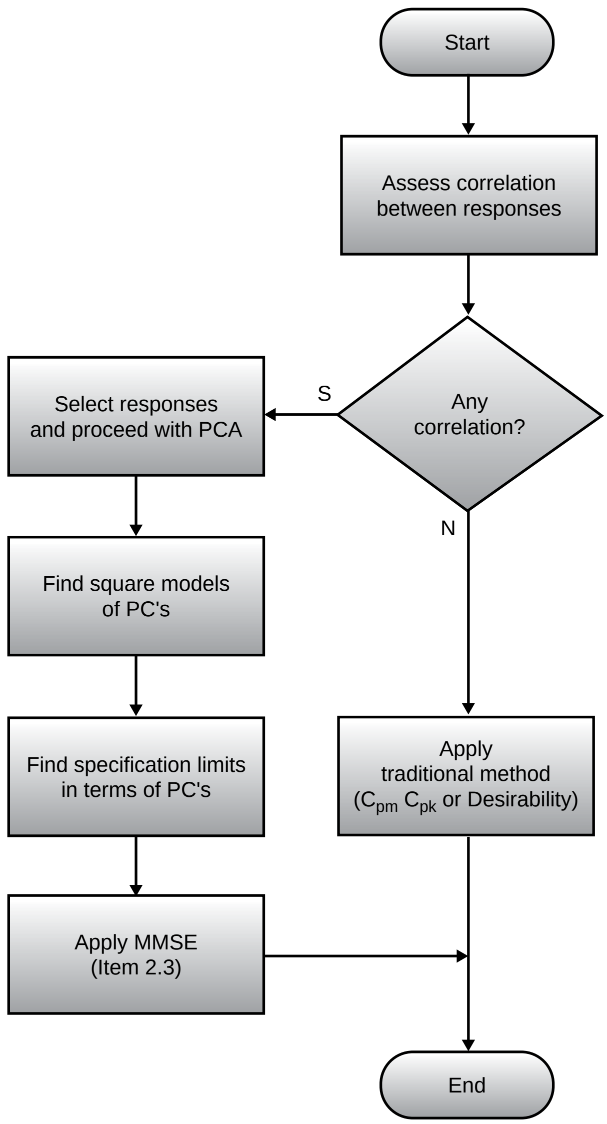


Figure 1. Flow of procedures to obtain MMSE.

### 2.1 Planning and analysis of experiments

The methodology of the DOE is one of the principal strategies available for the improvement of processes. Based on a systematized analysis of the problem, we sought to assess the magnitude of various sources of variation which influence a process. According to Montgomery[8], the process must begin with the identification and selection of factors which may contribute to a variation and then proceed with the selection of a model which includes the factors chosen, defining their levels and planning efficient experiments in order to estimate their effects.

Conducting experiments appropriately in accordance with the planning drawn-up ensures the success of the problem under study or at least avoids the risks of lack of success of unplanned experimentation. While conducting an experiment, one must be attentive to detect any abnormalities which occur as well as document these for subsequent analysis when the factors included in the model will be estimated, employing the appropriate statistical methods, culminating in inference, interpretation, and the discussion of results as well as the recommendation of improvements when necessary.

Once the factors and their respective levels are selected, a combination of these factors is generated in the form of experimental set-ups. The most common set-up is the complete factorial for which the number of experiments is equal to the number of experimental levels times the number of factors. In the typical case of factorials on two levels, the number of experiments is expressed by $N = 2^k$. Complete factorials cover the entire experimental range. Nevertheless, due to its exponential growth, set-ups with a large number of factors may make an experimental process unfeasible. For these cases, Montgomery and Runger[9] affirm that if there is little interest in the interactions, these may be neglected, resulting in fractions of the complete experiment without, however, compromising detection of the presence of influential factors.

The optimum point of a process is, however, generally associated with stationary points and only obtainable in surfaces or nonlinear functions (in general, curved, concave, convex, or mixed). Hence, a natural evolution of experimental practice is to migrate from factorial set-ups to RSM. RSM is a set of mathematical and statistical tools used to model and analyse problems which are influenced by numerous variables and for which square responses are wanted in general[8]. Generally, the relationship between dependent and independent variables is unknown and one looks to find a reasonable approximation of the true relationship between responses ($y$) and the set of independent variables ($x$). As any function even when unknown may be approximated by a Taylor series[8], the trunk of this series in its square term generates a nonlinear function known as 'response surface'. Therefore, if curvature exists in a system, then the approximation function most used for a second-order model is the one presented by Equation (1):

$$y = \beta_0 + \sum_{i=1}^{k} \beta_i x_i + \sum_{i=1}^{k} \beta_i x_i^2 + \sum_{i<j}\sum \beta_j x_i x_j + \varepsilon, \quad (1)$$

where $\beta$ is the polynomial coefficient, $k$ is the number of factors and $\varepsilon$ is the random error. The $\beta$ parameters of the model may be estimated using the ordinary square minimum method which in a matrix form may be represented by Equation (2):

$$\hat{\beta} = (X^{\mathrm{T}}X)^{-1}X^{\mathrm{T}}Y. \tag{2}$$

In Equation (2), $X$ is the matrix of codified factors and $Y$ is the response of interest.

The primary experimental set-up for the adjustment of response surfaces is the central composite design (CCD). According to Montgomery[8], part of this set-up is formed of a factorial arrangement (complete or fractioned) followed by the addition of central and axis points. Central points are equivalent to the mean points of the levels of quantitative factors, and axis points are distanced from the central ones at a constant ratio of $\rho = \sqrt[4]{2^k}$. This is the set-up used in this paper.

### 2.2 Surface of dual response

Multidimensional problems of the NTB type are those in which one seeks to minimize the distance between various responses ($Y_i$) and their respective targets ($\zeta_\gamma$), along with a reduction in their variances.

To achieve these goals, Vining and Myers[10] proposed the dual RSM as a way of achieving the goals proposed for each quality characteristic involved, based on a response surface for the mean ($\omega_\mu$) and another for variance ($\omega_{\sigma^2}$), both expressed as a second-order polynomial. Lin and Tu[7] proposed a combination of these two functions by minimizing the MSE as a criterion for the simultaneous optimization of the mean and variance, as shown in Equation (3):

$$\mathrm{MSE} = (\hat{\omega}_\mu - \zeta)^2 + \hat{\omega}_\sigma^2. \tag{3}$$

For multiple responses, however, two strategies can be adopted: (a) agglutination of the equations from the MSE of each response using their weighted sum or (b) selection of the equation of the MSE from the response of greatest importance as objective function, attributing restrictions to the remainder[6,11].

Derringer and Suich[12] proposed a set of transformations for each of the $p$ responses resulting in an individual function known as *Desirability* ($d_i$), where $0 \le d_i \le 1$. This method enables the individual importance of each response to be included ($w_i$). Though this was not specifically developed for dual response surface problems, the formula may be used with this connotation in problems of the NTB type. In this case, the *Desirability* transformation may be expressed as the system of Equation (4)[12]:

$$d_i[f_i(\mathbf{x})] = \begin{cases} 0 & \text{se} \quad f_i(\mathbf{x}) \le f_i^{\min} \text{ or } f_i(\mathbf{x}) > f_i^{\max} \\ \frac{f_i(\mathbf{x}) - f_i^{\min}}{T_i - f_i^{\min}} & \text{se} \quad f_i^{\min} < f_i(\mathbf{x}) \le T \\ \frac{f_i^{\max} - f_i(\mathbf{x})}{f_i^{\max} - T_i} & \text{se} \quad T_i < f_i(\mathbf{x}) \le f_i^{\max} \end{cases} \tag{4}$$

In the system of Equation (4), $f_i(\mathbf{x})$,$f_i^{\min}$,$f_i^{\max}$, and $T_i$ are, respectively, the value of $f_i(\mathbf{x})$ in the optimum, the lower limit of specification for the response, the upper limit of specification and the target (nominal value). For these problems, response variables are dealt within acceptance ranges. Individual transformations for responses may be combined using a geometric mean ($D$) as shown in Equation (5):

$$D = \left[\prod_{i=1}^{n} d_i^{w_i}(\hat{Y}_i)\right]^{1/W}. \tag{5}$$

The *Desirability* method, however, has several limitations, in particular, dependent on the method of a subjective choice of individual $d_i$ functions and negligence of the variances of responses, and the correlation structure between these[1,13]. In the example which shall be presented in this paper, though variables are not, in general, of the NTB type, this supposition shall be established. For example, penetration ($P$) of the weld bead is typically a response of the *Larger-is-better* type (maximization). However, for the purposes of this application, it shall be treated in terms of a tolerance or range.

### 2.3 Multivariate mean square error (MMSE)

Based on the MSE employed by Koksoy and Yalcinoz[6], Lin and Tu[7], Paiva[5], and Paiva et al.[14,15] proposed an adaptation capable of adequately incorporating the existing correlation structure between responses of interest, based on combinations of the RSM and PCA. Using this combination, an adjusted response surface is generated for the PC scores with which one then composes the MMSE.

PCA is a multivariate statistical technique capable of explaining the variance–covariance structure present in a set of data, using non-correlated linear combinations of the original variables, with the aim of reducing the size of the entry or exit vectors in the given equations[16]. This technique facilitates its interpretation since according to Rencher[17], it reveals relationships which would not be identified previously with the original set.

The basic idea of PCA is that though $p$ components are necessary to reproduce the total variability of a system of interest, in general, most of this variability may be represented by a small group of $p$ PCs. There is as much information in $p$ PCs as in $p$ original variables. Hence, the original set of data may be reduced to a few PCs.

The PCs depend solely on the variance–covariance matrix $\Sigma$ or correlation matrix $p$ of the variables $X_1, X_2, \ldots, X_p$. Or the random vector $X^{\mathrm{T}} = [X_1, X_2, \ldots, X_p]$, whose matrix of variance–covariance $\Sigma$ has auto values $\lambda_1 \ge \lambda_2 \ge \cdots \ge \lambda_p \ge 0$. The first $\mathrm{PC}_1$ is the linear combination which enables the maximum variance[16]. A set of original variables may be substituted by linear combinations in the form of scores of the PC.

The most commonly used criterion to determine the number of PCs which must be used is the Kaiser criterion[16], according to which the auto value of the PC must be higher than one to represent the original set. In addition to this, the accumulated variance explained must be superior to 80%.

Optimization based on MSE is represented by Equation (3). However, for multivariate problems with a single PC, this equation must be modified in accordance with Equation (6):

$$\text{Minimize}\,\text{MMSE}_{\text{PC}} = (\text{PC}_i - \zeta_{\text{PC}_i})^2 + \lambda_{\text{PC}_i} \tag{6}$$

$$\text{subject to} \quad x^{\text{T}}x \le \rho^2, \tag{7}$$

where $\text{PC}_i$ is the equation of the $i$th PC and is $\zeta_{\text{PC}_i}$ its target value, $\lambda_{\text{PC}_i}$ is the auto value of the $i$th PC and $x^Tx \le \rho^2$ is the restriction of the experimental space for spherical regions (in the event a CCD-type set-up is used for three factors, $\rho = 1.682$[8]). Finally, envisaging the optimization of various response surfaces for PCs we end up with

$$\text{Minimize}\left[\prod_{i=1}^{n}(\text{MMSE}_{\text{PC}_i}|\lambda_i \ge 1)\right]^{1/n} \tag{8}$$

$$\text{subject to} \quad x^{\text{T}}x \le \rho^2, \tag{9}$$

where $n$ is the number of MMSE functions considered in accordance with the significant PCs.

In this paper, the MMSE method will be compared with the traditional method of *Desirability* solely for target-type problems (NTB).

## 3. Optimization of FCAW process

Given that welding processes present many peculiar and specific characteristics, the MMSE method may be a relevant option in the search for optimization conditions. In this paper, the proposed approach was applied in optimization of a FCAW process, whose aim, originally investigated by Rodrigues[18], was the analysis of the ideal combination of welding parameters with rutilic flux-cored wires and their results in several specific welding characteristics. The test performed in which welds of straightforward depositions on ABNT 1045 steel plates measuring 75 mm × 50 mm × 9 mm were made use of, reverse polarity (CC+) using AWS E71T-1 wire of 1.2 mm in diameter, welding speed of 50 cm/min, and $CO_2$ shielding gas at a flow rate of 15 l/min and a torch inclination of 70°in the pulling direction, hence seeking to increase the penetration of the weld. A multiprocess Inversal 300 welding source was used with digital command and standard mode operation. The welding torch was hooked up to a car with speed adjustment and a mechanical system which enables a positioning adjustment pursuant to the conditions specified. To assess the length of wire consumed making each bead, a tachometer was used attached to the welding source which also enabled the time the arc was open to be assessed. Software to acquire voltage and current data was used in all the tests making it possible to observe the dynamic characteristics of drop transfer.

Taking into account factors such as voltage ($V$), wire feed speed (Va), and contact nozzle to part distance ($d$), the penetration ($P$), deposition rate (DR), yield ($Y$), convexity index (CI), and dilution ($D$) were registered.

DR (kg/h) was determined using Equation (10):

$$D = \frac{3.6(m_{\text{f}} - m_i)}{t}, \tag{10}$$

where DR is the deposition rate (kg/h), $m_{\text{f}}$ is the mass of test sample after welding (g), $m_i$ is the mass of test sample before welding (g) and $t$ is the welding time (s).

Yield ($Y$) was determined by the percentage ratio of the DR and fusion rate and expressed by Equation (11):

$$R = \frac{(m_{\text{f}} - m_i)}{d_L \times L} \times 100, \tag{11}$$

where $d_L$ is the linear density of the wire used (g/m) and $L$ the length of the cast wire (m).

After performing the welds, the test samples were cut into sections at two separate places in order to reduce measurement errors. Subsequently, these were polished and chemically etched with 4% Nital and assessed using an Olympus – Analysis Digital Imaging Solutions image analyser, the registered values $P$, $h$, $b$, $S_{\text{p}}$, $S_{\text{t}}$. CI, and $D$ were calculated, respectively, using Equations (12) and (13):

$$\text{CI} = \frac{h}{b} \times 100, \tag{12}$$

where $h$ is the bead reinforcement (mm) and $b$ is the bead width (mm):

$$D = \frac{S_{\text{p}}}{S_{\text{t}}} \times 100, \tag{13}$$

where $S_{\text{p}}$ is the area corresponding to bead penetration ($mm^2$) and $S_{\text{t}}$ is the area corresponding to total bead ($mm^2$).

Using a response surface set-up of the CCD type for three factors ($V$, Va, and $d$) as a basis for performing the experiments, the results given in Table 1 were obtained.

Table 1 also contains the current values measured during the welding process. It is worth pointing out that as a result of the source being the constant voltage type when regulating the voltage and wire feed speed parameters, the welding current adjusts itself in order to guarantee arc length and as a result the voltage value.

To assess whether the data of the FCAW process are appropriate for application of the MMSE method, an analysis was carried out of the correlation between the responses obtained as specified by the flow diagram in Figure 1. Using analysis of the correlation (Table 2), we noted that there was a strong correlation between response $P$ in relation to DR and $Y$, moderate correlation between $P$ and $D$, and no correlation between $P$ and CI, strong correlation of response DR in relation to $Y$, strong correlation of response CI in relation to $D$, and moderate correlation of DR in relation to CI. For all other responses, the correlations were either insignificant or inexistent. Hence, no correlations were analysed between the input variables (factors) because experimental set-ups, such as the CCD (Table 1), factorial set-ups, or Taguchi are

Table 1. Matrix of experiments (CCD) of the FCAW process and pre-processing of data.

| Information | | | Factors | | | Original responses | | | | | PCs | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Points | $n$ | $I$ (A) | $V$ | Va | $d$ | $P$ (mm) | TD (kg/h) | $R$ (%) | CI (%) | $D$ (%) | $PC_1$ | $PC_2$ | $PC_3$ |
| Complete factorial | 1 | 235 | 29.0 | 10.0 | 15.0 | 1.89 | 3.62 | 82.49 | 32.10 | 38.76 | −1.537 | −0.640 | −0.429 |
| | 2 | 246 | 36.0 | 10.0 | 15.0 | 1.93 | 3.49 | 79.47 | 21.68 | 44.26 | −2.874 | 0.499 | −0.948 |
| | 3 | 286 | 29.0 | 14.0 | 15.0 | 2.40 | 5.14 | 86.83 | 55.49 | 30.26 | 2.321 | −2.027 | −0.655 |
| | 4 | 300 | 36.0 | 14.0 | 15.0 | 2.88 | 5.28 | 86.03 | 26.10 | 45.13 | 1.263 | 1.727 | −0.136 |
| | 5 | 220 | 29.0 | 10.0 | 20.0 | 1.55 | 3.74 | 82.82 | 29.58 | 35.51 | −1.684 | −1.224 | 0.230 |
| | 6 | 228 | 36.0 | 10.0 | 20.0 | 1.94 | 3.65 | 81.99 | 25.52 | 43.23 | −1.932 | 0.269 | −0.370 |
| | 7 | 270 | 29.0 | 14.0 | 20.0 | 1.91 | 5.40 | 86.12 | 50.57 | 26.95 | 1.805 | −2.655 | −0.019 |
| | 8 | 212 | 36.0 | 14.0 | 20.0 | 2.85 | 4.52 | 85.58 | 24.70 | 43.67 | 0.473 | 1.489 | −0.174 |
| Axial points | 9 | 255 | 26.6 | 12.0 | 17.5 | 1.90 | 4.63 | 86.21 | 50.00 | 26.88 | 1.162 | −2.740 | 0.015 |
| | 10 | 276 | 38.4 | 12.0 | 17.5 | 2.40 | 4.59 | 84.10 | 23.80 | 42.11 | −0.248 | 0.863 | −0.066 |
| | 11 | 220 | 32.5 | 8.6 | 17.5 | 1.60 | 3.11 | 82.00 | 32.94 | 35.22 | −2.237 | −1.476 | −0.270 |
| | 12 | 319 | 32.5 | 15.4 | 17.5 | 3.40 | 6.06 | 89.12 | 44.21 | 44.37 | 3.842 | 1.417 | −0.652 |
| | 13 | 287 | 32.5 | 12.0 | 13.3 | 2.80 | 4.88 | 85.37 | 25.05 | 41.66 | 0.728 | 1.222 | −0.150 |
| | 14 | 259 | 32.5 | 12.0 | 21.7 | 1.90 | 4.65 | 86.42 | 23.75 | 35.05 | 0.087 | −0.426 | 1.303 |
| Central points | 15 | 262 | 32.5 | 12.0 | 17.5 | 1.90 | 4.55 | 86.05 | 27.27 | 42.43 | −0.116 | 0.250 | 0.865 |
| | 16 | 267 | 32.5 | 12.0 | 17.5 | 2.28 | 4.25 | 83.51 | 24.91 | 42.19 | −0.746 | 0.636 | −0.192 |
| | 17 | 262 | 32.5 | 12.0 | 17.5 | 2.30 | 4.37 | 84.82 | 26.85 | 42.65 | −0.236 | 0.657 | 0.078 |
| | 18 | 359 | 32.5 | 12.0 | 17.5 | 2.15 | 4.18 | 86.49 | 26.79 | 39.80 | −0.048 | 0.182 | 0.791 |
| | 19 | 260 | 32.5 | 12.0 | 17.5 | 2.30 | 4.22 | 85.79 | 25.06 | 44.30 | −0.217 | 0.947 | 0.428 |
| | 20 | 278 | 32.5 | 12.0 | 17.5 | 2.35 | 4.50 | 86.08 | 27.15 | 45.10 | 0.193 | 1.029 | 0.351 |
| Data processing for MMSE application | | | Column mean: Standard | | | 2.232 | 4.442 | 84.865 | 31.176 | 39.477 | 0.000 | 0.000 | 0.000 |
| | | | Target ($\zeta_Y$) $Z(Y_i\zeta_Y)$ | | | 0.468 | 0.714 | 2.225 | 10.211 | 5.884 | 1.623 | 1.369 | 0.548 |
| | | | deviation from column: | | | 2.700 | 4.650 | 85.000 | 26.000 | 40.000 | *−0.447* | *−0.829* | *0.251* |
| | | | | | | 1.000 | 0.292 | 0.061 | −0.507 | 0.089 | −0.276 | −0.606 | 0.458 |

Note: Values in italics represent targets calculated for PC scores.

Table 2. MMSE: correlation between responses of the FCAW process.

| | $P$ | DR | $Y$ | CI |
|---|---|---|---|---|
| DR | 0.721 | | | |
| | 0.000 | | | |
| $Y$ | 0.541 | 0.852 | | |
| | 0.014 | 0.000 | | |
| CI | − 0.027 | 0.447 | 0.416 | |
| | 0.909 | 0.048 | 0.068 | |
| $D$ | 0.491 | − 0.069 | − 0.104 | − 0.754 |
| | 0.028 | 0.773 | 0.662 | 0.000 |

orthogonal set-ups. This means that the correlation between input variables is null[8].

The correlation structure verified between the variables quoted makes application of the MMSE method feasible. Given the existence of the correlation between the responses, analysis was carried out of the PCs as specified in Table 3. From this analysis, we concluded that the three PCs explain more than 96% of the accumulated variance. Therefore, the MMSE method may be implemented using only three components. The values defined for the targets of the PCs ($\zeta\gamma$) as well as the values for the component scores (columns $PC_1$, $PC_2$, and $PC_3$ of Table 1) were calculated multiplying the auto vectors of each PC by the standardized value ($Z$) of each response of interest, as Johnson and Wichern[16] recommend. Both for the PCA and for generation and analysis of the experimental set-up of the response surface, the Minitab® 15.0 statistical software was used. The scores of the PCs and levels of variables shown in Table 1 are calculated directly by the software.

Once the responses obtained experimentally with the CCD set-up were computed, statistical analysis was carried out of the significance of the complete square models and the coefficients of the models determined as suggested in the flow diagram given in Figure 1. This analysis was carried out both for the original responses and for each of the PCs. Taking a significant level of $\alpha = 5\%$, all the complete square models presented in Table 4 were demonstrated to be statistically significant, though the adjustments obtained, in general, are very high. We would highlight, however, that the adjustment performed for the first PC obtained an adjusted $Y^2$ of almost 88%, demonstrating that analysis was made using the scores of this component which explains the majority of the variation observed in the five original responses. Both for the original responses and for the PCs, some terms did not present any individual significance; nevertheless, their removal from the models did not improve the adjustment or reduce the variability. Therefore, in accordance with the principle of hierarchy[8], the complete square models were maintained.

### 3.1 *Analysis of results*

To apply the MMSE and *Desirability* methods, the specification limits and targets shown in Table 5 were established, taking into account these conditions which were judged more appropriate for an optimized bead as well as information available in the literature[19]. It is worth pointing out that maximum yield was stipulated at 90% due to the wire being fluxed core and hence there were slag formations and spatters during the welding which compromised the yield. Targets were established as half of the tolerance.

Using the three component equations, MMSE can be formulated as a nonlinear optimization problem, such that

Minimize

$$\mathrm{MMSE}_T = \{[(\mathrm{PC}_1 - \zeta_{\mathrm{PC}_1})^2 + \lambda_1] < [(\mathrm{PC}_2 - \zeta_{\mathrm{PC}_2})^2 + \lambda_2] < [(\mathrm{PC}_3 - \zeta_{\mathrm{PC}_3})^2 + \lambda_3]\}^{1/3} \quad (14)$$

subject to

$$\mathbf{x}^{\mathrm{T}}\mathbf{x} \le \rho^2 = V^2 = \mathrm{Va}^2 = d^2 \le (1.682)^2 \quad (15)$$

with

$$\zeta_{\mathrm{PC}_1} = e_{1i}[Z(P|\zeta_{\mathrm{P}})] + e_{2i}[Z(\mathrm{TD}|\zeta_{\mathrm{TD}})] + e_{3i}[Z(R|\zeta_{\mathrm{R}})] + e_{4i}[Z(\mathrm{CI}|\zeta_{\mathrm{CI}})] + e_{5i}[Z(D|\zeta_{\mathrm{D}})] \quad (16)$$

$$i = 1, 2, \ldots, p, \quad (17)$$

where $Z$ represents the standardized value of $i$th response taking into account the target value $\zeta_Y$, such that

Table 3. Analysis of PCs of FCAW process and calculation of targets for components.

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| Auto values $\lambda_i$ | | | 2.5928 | 1.9075 | 0.3049 | 0.1301 | 0.0646 |
| Proportion (%) | | | 0.519 | 0.381 | 0.061 | 0.026 | 0.013 |
| Accumulated (% accum.) | | | 0.519 | 0.900 | 0.961 | 0.987 | 1.000 |
| Auto vectors of correlation matrix | | | | | | | |
| Variables | Targets | $Z$ (Targets) | $PC_1$ | $PC_2$ | $PC_3$ | $PC_4$ | $PC_5$ |
| $P$ | 2.70 | 1.0001 | − 0.427 | − 0.469 | 0.529 | − 0.145 | − 0.545 |
| DR | 4.65 | 0.2922 | − 0.600 | − 0.078 | 0.008 | − 0.441 | 0.663 |
| $Y$ | 85.00 | 0.0609 | − 0.566 | − 0.026 | − 0.701 | 0.314 | − 0.298 |
| CI | 26.00 | − 0.5069 | − 0.356 | 0.540 | 0.476 | 0.583 | 0.122 |
| $D$ | 40.00 | 0.0890 | 0.100 | − 0.694 | 0.039 | 0.589 | 0.400 |
| | | Targets ($\zeta_Y$) | − 0.4475 | − 0.8289 | 0.2509 | − 0.176 | 0.043 |

Table 4. FCAW: complete square models for responses.

| Term | $P$ | DR | $Y$ | CI | $D$ | $PC_1$ | $PC_2$ | $PC_3$ |
|---|---|---|---|---|---|---|---|---|
| $B_0$ | 2.219 | 4.355 | 85.509 | 26.354 | 42.705 | −0.167 | 0.620 | 0.397 |
| $V$ | 0.197 | −0.075 | −0.640 | −8.333 | 5.157 | −0.465 | 1.215 | −0.065 |
| Va | 0.422 | 0.791 | 2.179 | 4.901 | −0.026 | 1.765 | 0.329 | −0.008 |
| $d$ | −0.173 | −0.044 | 0.253 | −0.526 | −1.477 | −0.116 | −0.326 | 0.313 |
| $V^2$ | −0.059 | 0.027 | −0.450 | 3.629 | −2.656 | 0.047 | −0.567 | −0.214 |
| $Va^2$ | 0.065 | 0.018 | −0.306 | 4.222 | −0.782 | 0.169 | −0.246 | −0.368 |
| $d^2$ | 0.012 | 0.082 | −0.188 | −0.790 | −1.291 | 0.029 | −0.095 | −0.001 |
| $V \times$ Va | 0.124 | −0.065 | 0.314 | −5.098 | 2.296 | −0.101 | 0.658 | 0.185 |
| $V \times d$ | 0.101 | −0.123 | 0.306 | 1.235 | 0.509 | 0.102 | 0.093 | −0.094 |
| Va $\times d$ | −0.024 | −0.098 | −0.501 | −0.955 | −0.061 | −0.263 | −0.007 | −0.080 |
| $Y^2$ adj. (%) | 78.6 | 86.3 | 70.0 | 96.8 | 70.0 | 87.7 | 79.7 | 51.4 |

Table 5. Specification limits for parameters of FCAW process.

| Responses | Minimum | Target | Maximum |
|---|---|---|---|
| Penetration (mm) | 1.9 | 2.7 | 3.5 |
| DR (kg/h) | 3.8 | 4.6 | 5.5 |
| Yield (%) | 80 | 85 | 90 |
| CI (%) | 22 | 26 | 30 |
| Dilution (%) | 35 | 40 | 45 |

$Z(Y_i|\zeta_Y) = [(\zeta_Y) - \mu_Y](\sigma_Y)^{-1}$. The numeric values of the standardized targets $Z(Y_i|\zeta_Y)$ are mentioned in the last line of Table 3. The results obtained are demonstrated in Table 7. This system of nonlinear optimization may be resolved using a number of algorithms and software systems. In this paper, the generalized reduced gradient algorithm was used, available in MS-Excel Solver. The results of the *Desirability* method were obtained with a Minitab® 15.0 Response Optimizer though these can also be easily obtained with Solver.

Note that the solution presented via MMSE (in bold) achieves values within the specification limits for all the responses. *Desirability*, however, exceeds the maximum limits for $P$ (penetration) responses, DR and $D$ and the minimum limit for response CI. These results point to a substantial improvement in the solution when MMSE is employed, demonstrating its efficiency in simultaneous optimization and ratifying the good adaptation of the proposal to the simultaneous optimization of multiple characteristics. An outline graph was generated for the results produced using MMSE in order to verify the location of the optimum value in relation to responses obtained by the model, and compared with the results obtained for *Desirability*, as shown in Figure 2.

### *3.2 Confirmation experiments*

To assess the quality of the results obtained by the two methods, confirmation experiments were carried out with each of the two solutions encountered. The results are summarized in Table 7.

From Table 7, one concludes that the MMSE method provides closer true and theoretical values and, in general, with shorter distances in relation to the target value, demonstrating its efficacy in problems of the nominal type with correlated response variables.

The confirmation experiments enable us to comparatively assess the MMSE and *Desirability* methods, expressing the percentage errors observed between the theoretical values obtained through application of each one of the models and their actual values and the actual values in relation to their targets. The low-percentage errors seen denote the reliability of the MMSE model in the search for ideal process parameters.

## 4. Conclusion

This paper presented the optimization method for correlated multiple responses known as MMSE. This method may be employed in all processes which have NTB-type characteristics, with output characteristics which have a strong or moderate correlation structure. As may be observed in relation to the FCAW process, MMSE enabled attendance of the limits specified for all the responses without exceeding the experimental space restriction imposed, while we considered the correlation structure appropriate, presenting percentage errors in relation to confirmation values and in relation to the target which were satisfactorily minor, in detriment of the results of the *Desirability* method. Though the results

Table 6. Comparison between MMSE and *Desirability* methods.

| | $V$ | Va (m/min) | $d$ (mm) | $P$ (mm) | DR (kg/h) | $Y$ (%) | CI (%) | $D$ (%) |
|---|---|---|---|---|---|---|---|---|
| Minimum | 26.6 | 8.6 | 13.3 | 1.90 | 3.80 | 80.00 | 22.00 | 35.00 |
| Maximum | 38.4 | 15.4 | 21.7 | 3.50 | 5.50 | 90.00 | 30.00 | 45.00 |
| Desirability | *37.9* | *14.3* | *13.3* | 3.53 | 5.79 | 86.24 | 21.61 | 49.77 |
| MMSE | *32.2* | *12.4* | *13.6* | 2.62 | 4.76 | 84.96 | 27.67 | 41.45 |

Table 7. Comparative analysis of confirmation experiments.

| Method | | *P* | DR | *Y* | CI | *D* |
|---|---|---|---|---|---|---|
| | Targets | 2.70 | 4.65 | 85.00 | 26.00 | 40.00 |
| Desirability | Actual | 2.65 | 4.95 | 83.71 | 24.4 | 45.05 |
| | Theoretical | 3.53 | 5.79 | 86.25 | 21.61 | 49.77 |
| | Error (actual × theoretical) (%) | − 24.90 | − 14.50 | − 2.94 | 12.90 | − 9.48 |
| | Error (actual × target) (%) | − 0.05 | 0.30 | − 1.29 | − 1.60 | 5.05 |
| MMSE | Actual | 2.5 | 4.5 | 84.01 | 28.1 | 40.5 |
| | Theoretical | 2.62 | 4.76 | 84.96 | 27.67 | 41.45 |
| | Error (actual × theoretical) (%) | − 4.60 | − 4.50 | − 1.12 | 1.60 | − 2.20 |
| | Error (actual × target) (%) | − 0.20 | − 0.15 | − 0.99 | 2.10 | 0.50 |

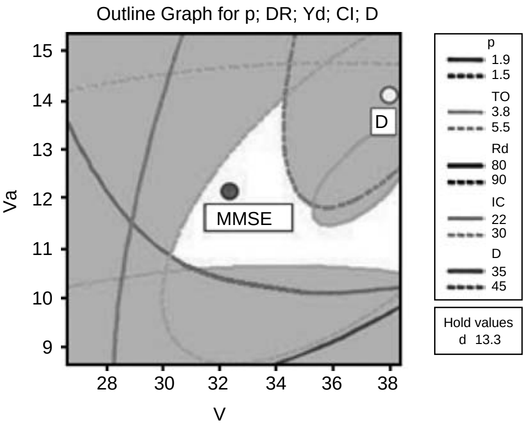


Figure 2. Outline graph for MMSE of the FCAW process. Key: (D) *Desirability*; (MMSE) multivariate method.

obtained in this study cannot be extrapolated or generalized, its application in optimization problems of manufacturing processes with multiple characteristics may be recommended.

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