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Weighted approach for multivariate analysis of variance in measurement system analysis

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ABSTRACT

In a process that is integral to a measurement system, some variation is likely to occur. Measurement system analysis is an important area of study that is able to determine the amount of variation. In evaluating a measurement system's variation, the most adequate technique, once an instrument is calibrated, is gauge repeatability and reproducibility (GR&R). For evaluating multivariate measurement systems, however, discussion has been scarce. Some researchers have applied multivariate analysis of variance to estimate the evaluation indexes; here the geometric mean is used as an agglutination strategy for the eigenvalues extracted from variance–covariance matrices. This approach, however, has some weaknesses. This paper thus proposes new multivariate indexes based on four weighted approaches. Statistical analysis of empirical and data from the literature indicates that the most effective weighting strategy in multivariate GR&R studies is based on an explanation of the percentages of the eigenvalues extracted from a measurement system' matrix.

1. Introduction

To properly monitor and improve a manufacturing process, it is necessary to measure attributes of the process's output. For any group of measurements collected for this purpose, at least part of the variation is due to the measurement system itself. This is because repeated measurements of any particular item occasionally result in different values [1–7]. To ensure that measurement system variability is not detrimentally large, it is necessary to conduct measurement system analysis (MSA). Such a study can be conducted in virtually any type of manufacturing industry. MSA helps to quantify the ability of a gauge or measuring device to produce data that supports analyst's decision-making requirements [8]. The purpose of this study is to (i) determine the amount of variability in collected data that is due to the measurement system, (ii) isolate the sources of variability in the measurement system, and (iii) assess whether the measurement system is suitable for use in a broader project or other applications [9,10]. According to He et al. [11], MSA is an important element of Six Sigma as well as of the ISO/TS 16949 standards.

The most common study in MSA to evaluate the precision of measurement systems is gauge repeatability and reproducibility (GR&R). Repeatability represents the variability from the gauge or measurement instrument when it is used to measure the same unit (with the same operator or setup or in the same time period). Reproducibility reflects the variability arising from different operators, setups, or time periods [7,10,12–17]. Some works in the literature [18–21] have used repeatability and/or reproducibility concepts; these, however, ignored GR&R statistical analysis in comparing measurement system variation to process variation. These studies involving only gauge variability are insufficient to determine whether the measurement system is able to monitor a particular manufacturing process. If variation due to the measurement system is small relative to the variation of the process, then the measurement system is deemed capable. This means the system can be used to monitor the process [9]. GR&R studies must be performed any time a process is modified. This is because as process variation decreases, a once-capable measurement system may now be incapable. Two methods commonly used in the analysis of a GR&R study are: (1) analysis of variance (ANOVA) and (2) Xbar and R chart [5,10]. Analysts prefer the ANOVA method because it measures the operator-to-part interaction gauge error—a variation not included in the Xbar and R method [4].

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Currently, the ANOVA method for GR&R studies can be applied only to univariate data [5,22]. To discriminate among products, however, manufacturers often use more than a single measurement on a single product characteristic [9]. To estimate evaluation indexes in such a GR&R study, the analyst must consider the correlation structure among the characteristics, a task more suited to multivariate methods [7]. Using automotive body panel gauge-study data, Majeske [3] demonstrated how to fit multivariate analysis of variance (MANOVA) model and estimate the evaluation indexes to multivariate measurement systems. In his analysis, it was shown that the multivariate approach had resulted in a more practical representation of the errors and led the manufacturer to approve the gauge. Wang and Yang [22] presented a GR&R study with multiple characteristics using principal component analysis (PCA). The authors pointed out that when correlated quality characteristics are present a GR&R study must be conducted carefully. In this case study, the composite indexes *P*/*T* (precision-to-tolerance) and *%R&R* (percentage of repeatability and reproducibility) with ANOVA method were overestimated by the PCA by 35.75% and 11.54%, respectively. Wang and Chien [5] analyzed a measurement system using a multivariate GR&R study and provided the confidence interval for two measures *P*/*T* and the number of distinct categories (ndc). Through a case study, the authors assessed the performance of three methods (ANOVA, PCA and POBREP—process-oriented basis representation). The authors argued that POBREP outperformed the others by being able to identify the causes of production problems. Peruchi et al. [7] proposed a multivariate GR&R method based on weighted PCA. The method was applied to experimental and simulated data to compare its performance to univariate and multivariate methods. The authors demonstrated that their weighted principal component (WPC) method was more robust than the others, considering not only several correlation structures but also distinct measurement systems.

Larsen [23] extended the univariate GR&R study to a common manufacturing test scenario where multiple characteristics were tested on each device. Illustrating with examples from an industrial application, the author showed that total yield, false failures, and missed false estimates could lead to improvements in the production test process and hence to lower production costs and, ultimately, to customers receiving higher quality products. Flynn et al. [24] used regression analysis to analyze the comparative performance capability between two functionally equivalent but technologically different automatic measurement systems. For such accurate measurements as repeatability and reproducibility, the authors found as inappropriate the "pass/fail" criteria for the unit being tested. Hence, they proposed a methodology based on PCA and MANOVA to examine whether there was a statistically significant difference among the measurement systems. He et al. [11] proposed a PCA-based approach in MSA for the in-process monitoring of all instruments in multisite testing. The approach considers a faulty instrument to be one whose statistical distribution of measurements differs significantly from the overall distribution across multiple test instruments. Their approach can be implemented as an online monitoring technique for test instruments so that, until a faulty instrument is identified, production goes uninterrupted. Parente et al. [25] applied univariate and multivariate methods to evaluate repeatability and reproducibility of the measurement of reverse phase chromatography (RP-HPLC) peptide profiles of extracts from cheddar cheese. The ability to discriminate different samples was assessed according to the sources of variability in their measurement and analysis procedure. The authors showed that their study had an important impact on the design and analysis of experiments for the profiling of cheese proteolysis. Inferential statistical techniques helped them analyze the relationships between design variables and proteolysis.

This paper focuses on multivariate analysis of variance method applied to GR&R studies (Section 2). The relevance of this topic lies in the fact that the variation of more complex measurement systems must be evaluated by more sophisticated methods. When multiple correlated characteristics are being monitored, multivariate analysis of variance can be applied to more precisely assess a measurement system. For calculating a multivariate evaluation index, however, a limitation can be found with the geometric mean strategy. To estimate the multivariate evaluation index, no attempt was made to quantify the greater importance to the most significant pair of eigenvalues, extracted from variance–covariance matrices for process, measurement system, and total variation. Therefore, the aim of this research is to come up with solutions to this problem by adopting weighted approaches to estimate the multivariate evaluation index (Section 3). The problem statement in this paper has been raised while assessing correlated roughness parameters from the AISI 12L14 turning process (Sections 4 and 5). Due to distinct estimates among the multivariate indexes, the authors have also included more numerical examples from the literature to show how the new proposed indexes obtained better accuracy (Section 6). Based on the large data set analyzed, the authors concluded that the weighted approaches using the explanation percentages of the eigenvalues extracted from measurement system matrix were the most appropriate strategy for multivariate GR&R studies assessed by multivariate analysis of variance (Section 7).

2. Measurement system analysis by multivariate GR&R study

When reporting the result of a measurement of a physical quantity, it is required that some quantitative indication of the quality of the result be given to assess its reliability. Measurement Uncertainty is a term that is used internationally to describe the quality of a measurement value. In essence, uncertainty is the value assigned to a measurement result that describes, within a defined level of confidence, the range expected to contain the true measurement result [26,27]. AIAG [4] states that the major difference between uncertainty and the MSA is that the MSA focus is on understanding the measurement process to promote improvements (variation reduction). MSA determines the amount of error in the process and assesses the adequacy of the measurement system for product and process control. MSA applies statistical techniques to quantify process and measurement system components of variation. A general ANOVA model in MSA is represented by Eq. (1) [4,8,9,17,28]:

$$Y = X + E \tag{1}$$

In this expression, Y is the measured value of a randomly selected part from a manufacturing process, X is the true value of the part, and E is the measurement error attributed to the measurement system. The terms X and E are independent normal random variables with means μ_p and μ_{ms} and variances σ_p^2 and σ_{ms}^2, respectively. The mean μ_{ms} is referred to as the measurement system's bias. Typically, this bias can be eliminated by proper calibration of the system [9]. Thus, if it is assumed $\mu_{ms} = 0$, it can also be concluded that $\mu_Y = \mu_p$. If this assumption is violated, it will affect the estimation of μ_p but not the estimation of the variances. Since in a GR&R study the variances are of primary interest, the ANOVA model with p parts, o operators and r replicates can be expanded to Eq. (2) [6,7,17,29]:

$$Y = \mu + \alpha_i + \beta_j + (\alpha\beta)_{ij} + \varepsilon_{ijk} \begin{cases} i = 1, 2, \ldots, p \\ j = 1, 2, \ldots, o \\ k = 1, 2, \ldots, r \end{cases} \tag{2}$$

In this expression, μ is the mean of the measured value (assuming $\mu_{ms} \cong 0$ and $\mu_Y = \mu_p$ as mentioned above), α_i is the variance for process, β_j is the variance for operators, $(\alpha\beta)_{ij}$ is the variance for process and operators interaction, and ε_{ijk} is the variance for repeatability (random error). The ANOVA method for GR&R studies can be applied to one single quality characteristic only. For measurement systems that are capable of measuring q correlated Ys, the model in Eq. (2) turns into a matrix representation, through multivariate analysis of variance, where $\mathbf{Y} = (\mathbf{Y}_1, \mathbf{Y}_2, \ldots, \mathbf{Y}_q)$ and $\boldsymbol{\mu} = (\boldsymbol{\mu}_1, \boldsymbol{\mu}_2, \ldots, \boldsymbol{\mu}_q)$ are constant vectors; $\boldsymbol{\alpha}_i \sim N(\mathbf{0}, \boldsymbol{\Sigma}_\alpha)$, $\boldsymbol{\beta}_j \sim N(\mathbf{0}, \boldsymbol{\Sigma}_\beta)$, $\boldsymbol{\alpha\beta}_{ij} \sim N(\mathbf{0}, \boldsymbol{\Sigma}_{\alpha\beta})$, and $\boldsymbol{\varepsilon}_{ijk} \sim N(\mathbf{0}, \boldsymbol{\Sigma}_\varepsilon)$ are random vectors statistically independent of each other. To estimate the variance–covariance matrices in Eq. (2), mean squares matrices **MSP** (process variation), **MSO** (operator), **MSPO** (process × operator interaction) and **MSE** (random error term) are calculated as such, Eqs. (3)–(6):

$$\mathbf{MSP}_{YaYb} = \frac{or}{p-1}\sum_{i=1}^{p}(\bar{Y}_{a_{i..}} - \bar{Y}_{a_{...}})(\bar{Y}_{b_{i..}} - \bar{Y}_{b_{...}})' \tag{3}$$

$$\mathbf{MSO}_{YaYb} = \frac{pr}{(o-1)}\sum_{j=1}^{o}(\bar{Y}_{a_{.j.}} - \bar{Y}_{a_{...}})(\bar{Y}_{b_{.j.}} - \bar{Y}_{b_{...}})' \tag{4}$$

$$\mathbf{MSPO}_{YaYb} = \frac{r}{(p-1)(o-1)}\sum_{i=1}^{p}\sum_{j=1}^{o}(\bar{Y}_{a_{ij.}} - \bar{Y}_{a_{i..}} - \bar{Y}_{a_{.j.}} + \bar{Y}_{a_{...}}) \times (\bar{Y}_{b_{ij.}} - \bar{Y}_{b_{i..}} - \bar{Y}_{b_{.j.}} + \bar{Y}_{b_{...}})' \tag{5}$$

$$\mathbf{MSE}_{YaYb} = \frac{1}{po(r-1)}\sum_{i=1}^{p}\sum_{j=1}^{o}\sum_{k=1}^{r}(Y_{a_{ijk}} - \bar{Y}_{a_{...}})(Y_{b_{ijk}} - \bar{Y}_{b_{...}})' \tag{6}$$

In these expressions, the subscript Y_aY_b refers to the pair of variables for calculating mean square matrices, Ya_{ijk} represents the kth reading by jth operator of the ith part for the measurand a. $\bar{Y}a_{...} = \sum_{i=1}^{p}\sum_{j=1}^{o}\sum_{k=1}^{r} Ya_{ijk}/por$, $\bar{Y}a_{i..} = \sum_{j=1}^{o}\sum_{k=1}^{r} Ya_{ijk}/or$, $\bar{Y}a_{.j.} = \sum_{i=1}^{p}\sum_{k=1}^{r} Ya_{ijk}/pr$, and $\bar{Y}a_{ij.} = \sum_{k=1}^{r} Ya_{ijk}/r$ indicate averages taken across the subscripts replaced by a period [30–32]. Then, the variance–covariance matrices are estimated for process ($\hat{\boldsymbol{\Sigma}}_p$), reproducibility ($\hat{\boldsymbol{\Sigma}}_{reproducibility}$), repeatability ($\hat{\boldsymbol{\Sigma}}_{repeatability}$), measurement system ($\hat{\boldsymbol{\Sigma}}_{ms}$) and total variation ($\hat{\boldsymbol{\Sigma}}_t$), according to Eqs. (7)–(11):

$$\hat{\boldsymbol{\Sigma}}_p = \hat{\boldsymbol{\Sigma}}_\alpha = \frac{\mathbf{MSP} - \mathbf{MSPO}}{or} \tag{7}$$

$$\hat{\Sigma}_{reproducibility} = \hat{\Sigma}_\beta + \hat{\boldsymbol{\Sigma}}_{\alpha\beta} = \frac{\mathbf{MSO} - \mathbf{MSPO}}{pr} + \frac{\mathbf{MSPO} - \mathbf{MSE}}{r} \tag{8}$$

$$\hat{\boldsymbol{\Sigma}}_{repeatability} = \hat{\boldsymbol{\Sigma}}_\varepsilon = \mathbf{MSE} \tag{9}$$

$$\hat{\Sigma}_{ms} = \hat{\boldsymbol{\Sigma}}_{repeatability} + \hat{\Sigma}_{reproducibility} \tag{10}$$

$$\hat{\Sigma}_t = \hat{\boldsymbol{\Sigma}}_p + \hat{\boldsymbol{\Sigma}}_{ms} \tag{11}$$

If the interaction effect is not significant, the model of Eq. (2) can be reduced to the model of Eq. (12). In this case, the mean square matrix of **MSE** is estimated by Eq. (13) and variance–covariance matrices for part (process) and reproducibility (operator) are estimated using, respectively, Eqs. (14) and (15). **MSP**, **MSO**, $\hat{\boldsymbol{\Sigma}}_{repeatability}$, $\hat{\boldsymbol{\Sigma}}_{ms}$ and $\hat{\boldsymbol{\Sigma}}_t$ are estimated using previously mentioned Eqs. (3), (4), (9), (10) and (11), respectively.

$$\mathbf{Y} = \boldsymbol{\mu} + \boldsymbol{\alpha}_i + \boldsymbol{\beta}_j + \boldsymbol{\varepsilon}_{ijk} \begin{cases} i = 1, 2, \ldots, p \\ j = 1, 2, \ldots, o \\ k = 1, 2, \ldots, r \end{cases} \tag{12}$$

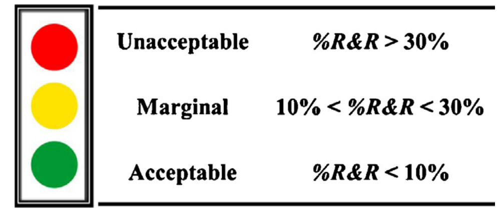

Fig. 1. GR&R criteria for measurement system acceptability.

$$\mathbf{MSE}_{YaYb} = \frac{1}{por - p - o + 1}\sum_{i=1}^{p}\sum_{j=1}^{o}\sum_{k=1}^{r}(Y_{a_{ijk}} - \bar{Y}_{a_{i..}} - \bar{Y}_{a_{.j.}} + \bar{Y}_{a_{...}}) \times (Y_{b_{ijk}} - \bar{Y}_{b_{i..}} - \bar{Y}_{b_{.j.}} + \bar{Y}_{b_{...}})' \tag{13}$$

$$\hat{\Sigma}_p = \hat{\boldsymbol{\Sigma}}_\alpha = \frac{\mathbf{MSP} - \mathbf{MSE}}{or} \tag{14}$$

$$\hat{\Sigma}_{reproducibility} = \hat{\Sigma}_\beta = \frac{\mathbf{MSO} - \mathbf{MSE}}{pr} \tag{15}$$

The multivariate version of the %R&R index with q characteristics proposed by Majeske [3] is called here G index and is calculated by Eq. (16). $\boldsymbol{\lambda}_{ms_i}$ and $\boldsymbol{\lambda}_{t_i}$ $\forall i = 1, 2, \ldots, q$ are eigenvalues extracted from variance–covariance matrices, $\boldsymbol{\Sigma}_{ms}$ and $\boldsymbol{\Sigma}_t$:

$$G = \left(\prod_{i=1}^{q}\sqrt{\frac{\boldsymbol{\lambda}_{ms_i}}{\boldsymbol{\lambda}_{t_i}}}\right)^{1/q} \times 100\% \tag{16}$$

For measurement systems whose purpose is to analyze a process, a general guideline for measurement system acceptability is presented in Fig. 1. If the measurement system, according to the index, is <10%, then it is considered acceptable. If it is between 10% and 30%, then it is considered marginal—acceptable depending on the application, the cost of the measurement device, the cost of repair and other factors. If the measurement system exceeds 30%, then it is considered unacceptable and should be improved [3,5,7].

3. Multivariate GR&R study using weighted approaches

To obtain the evaluation index to the measurement system, Majeske [3] applied geometric mean to the ratio $\sqrt{\lambda_{ms}/\lambda_t}$. To estimate G index, no attempt was made to quantify the greater importance to the most significant pair of eigenvalues, extracted from variance-covariance matrices. A more accurate index would determine a strategy of weighting to take into consideration the most significant information concerning the $\sqrt{\lambda_{ms}/\lambda_t}$ ratios and accordingly refine the multivariate index estimation. As a result, this article adopts weighted approaches upon $\sqrt{\lambda_{ms}/\lambda_t}$ ratio that are based on geometric and arithmetic means to propose four new evaluation indexes for multivariate measurement systems. These new indexes, WA_t, WA_{ms}, WG_t and WG_{ms}, can be obtained based on Eqs. (17) and (18):

$$WA = \sum_{i=1}^{q}\left(\mathbf{W}_i\sqrt{\frac{\boldsymbol{\lambda}_{ms_i}}{\boldsymbol{\lambda}_{t_i}}}\right) \times 100\% \tag{17}$$

$$WG = \prod_{i=1}^{q}\left(\sqrt{\frac{\boldsymbol{\lambda}_{ms_i}}{\boldsymbol{\lambda}_{t_i}}}\right)^{\mathbf{W}_i} \times 100\% \tag{18}$$

In this expression, $W_i\ \forall\, i = 1, \ldots, q$ are the explanation percentage of the eigenvalues extracted from either $\hat{\Sigma}_t : W_i = (\lambda_{t_i}/\sum_{j=1}^{q}\lambda_{t_j})$ or $\hat{\Sigma}_{ms} : W_i = (\lambda_{ms_i}/\sum_{j=1}^{q}\lambda_{ms_j})$. The WA_t and WA_{ms} indexes are

Table 1
Roughness measured for the turning process of mild steel.

		$k=1$					$k=2$					$k=3$					$k=4$				
j	i	R_a	R_y	R_z	R_q	R_t	R_a	R_y	R_z	R_q	R_t	R_a	R_y	R_z	R_q	R_t	R_a	R_y	R_z	R_q	R_t
1	1	1.41	8.65	7.14	1.76	8.86	1.50	8.97	7.55	1.82	9.19	1.53	8.06	6.73	1.77	8.06	1.46	7.08	6.47	1.72	8.28
1	2	1.42	10.05	7.13	1.74	10.05	1.30	9.11	6.69	1.66	9.49	1.35	8.64	7.44	1.67	9.05	1.39	9.47	7.07	1.69	9.47
1	3	1.42	9.47	7.47	1.76	9.47	1.51	8.89	7.42	1.88	9.43	1.22	7.19	6.40	1.50	7.48	1.40	9.01	7.23	1.77	9.46
1	4	1.81	9.74	7.66	2.08	9.74	1.77	9.51	7.92	2.07	10.06	1.72	7.28	6.63	1.95	7.48	1.92	10.67	8.22	2.23	11.07
1	5	2.06	7.40	7.03	2.28	7.58	2.06	7.26	6.96	2.26	7.56	2.08	7.10	6.97	2.30	7.22	2.03	7.38	7.04	2.26	7.43
1	6	2.20	7.81	7.46	2.41	7.92	2.17	7.85	7.34	2.39	7.85	2.19	7.49	7.28	2.41	7.77	2.15	8.06	7.38	2.38	8.06
1	7	1.75	8.44	8.38	2.12	8.89	1.97	10.30	9.73	2.53	10.31	1.72	8.31	7.77	2.06	8.58	2.07	9.85	9.65	2.56	9.96
1	8	0.81	5.36	4.33	1.01	5.36	0.79	4.29	3.91	0.98	4.30	0.84	5.54	4.12	1.03	5.54	0.79	4.54	4.04	1.00	4.61
1	9	0.77	4.41	3.91	0.96	4.55	0.79	5.14	4.18	1.00	5.15	0.84	4.42	3.95	1.04	4.42	0.84	5.58	4.76	1.31	5.89
1	10	1.83	9.29	8.36	2.20	9.72	1.62	8.47	7.11	1.92	9.08	1.84	8.91	7.80	2.16	9.26	1.96	9.78	8.00	2.26	10.60
1	11	1.73	7.17	6.50	2.00	7.17	1.72	7.61	6.69	1.99	7.63	1.75	7.61	6.75	2.02	7.61	1.71	7.08	6.50	1.98	7.38
1	12	2.00	8.31	7.72	2.28	8.65	1.83	7.80	7.24	2.11	7.83	1.86	7.98	7.33	2.15	8.19	1.89	8.06	7.48	2.17	8.34
2	1	1.40	8.67	7.28	1.76	8.91	1.55	8.89	7.55	1.87	8.95	1.52	8.68	6.72	1.85	8.68	1.44	6.94	6.39	1.70	7.90
2	2	1.41	10.04	7.20	1.74	10.19	1.31	9.12	6.69	1.67	9.51	1.36	9.13	6.88	1.66	9.37	1.41	10.00	6.71	1.68	10.60
2	3	1.43	9.56	7.54	1.76	9.61	1.51	8.87	7.40	1.88	9.37	1.18	7.10	6.31	1.47	7.70	1.44	9.00	7.70	1.84	9.46
2	4	1.84	10.47	8.09	2.14	10.80	1.78	9.37	7.97	2.07	9.99	1.70	7.34	6.68	1.94	7.57	1.93	9.93	8.22	2.24	11.41
2	5	2.06	7.24	7.01	2.27	7.43	2.06	7.31	6.97	2.27	7.53	2.08	7.09	7.00	2.30	7.29	2.07	7.38	7.21	2.30	7.70
2	6	2.20	7.76	7.43	2.41	7.95	2.17	7.82	7.33	2.39	7.82	2.19	7.48	7.28	2.41	7.83	2.14	8.02	7.41	2.37	8.02
2	7	1.75	8.42	8.34	2.12	8.89	1.97	10.00	9.62	2.53	10.07	1.72	8.44	7.84	2.07	8.69	2.06	9.78	9.64	2.56	9.94
2	8	0.82	5.51	4.34	1.02	5.51	0.76	4.28	3.91	0.96	4.28	0.85	5.17	4.19	1.05	5.19	0.78	4.57	4.09	0.99	4.67
2	9	0.78	4.39	3.96	0.98	4.39	0.79	4.40	4.05	0.99	4.57	0.84	4.41	4.00	1.03	4.59	0.84	5.55	4.71	1.30	5.77
2	10	1.84	9.32	8.36	2.21	9.74	1.62	8.62	7.10	1.92	9.16	1.83	8.83	7.76	2.16	9.13	1.96	9.34	7.94	2.26	10.22
2	11	1.74	7.11	6.49	2.00	7.11	1.72	7.69	6.69	2.00	7.69	1.74	7.48	6.76	2.01	7.53	1.71	7.08	6.52	1.98	7.40
2	12	2.00	8.30	7.67	2.28	8.59	1.84	7.81	7.25	2.11	7.83	1.89	7.94	7.42	2.18	8.34	1.90	8.11	7.49	2.17	8.35
3	1	1.41	8.66	7.38	1.76	8.89	1.56	8.45	7.46	1.88	8.90	1.52	8.40	6.72	1.80	8.40	1.43	7.02	6.40	1.70	7.96
3	2	1.42	10.08	7.21	1.75	10.85	1.30	9.10	6.73	1.66	9.56	1.48	8.40	7.10	1.78	9.70	1.66	9.42	7.65	1.96	9.42
3	3	1.43	9.31	7.45	1.76	9.31	1.51	8.96	7.48	1.88	9.47	1.22	7.54	6.54	1.53	8.29	1.44	9.01	7.68	1.84	9.35
3	4	1.84	10.29	8.13	2.14	10.65	1.78	9.37	7.99	2.07	10.01	1.70	7.37	6.67	1.94	7.59	1.94	10.66	8.51	2.26	11.65
3	5	2.06	7.40	7.05	2.28	7.59	2.06	7.29	6.98	2.27	7.59	2.08	7.11	6.98	2.30	7.31	2.08	7.37	7.18	2.30	7.59
3	6	2.19	7.81	7.47	2.41	8.05	2.16	7.78	7.32	2.39	7.78	2.19	7.50	7.25	2.41	7.83	2.15	7.99	7.40	2.38	7.99
3	7	1.75	8.41	8.34	2.12	8.86	1.97	9.97	9.66	2.53	10.04	1.72	8.40	7.83	2.07	8.74	2.06	9.84	9.67	2.55	10.00
3	8	0.82	5.68	4.41	1.03	5.68	0.77	4.14	3.81	0.96	4.14	0.85	5.15	4.16	1.05	5.15	0.77	4.59	4.05	0.98	4.74
3	9	0.77	4.55	4.01	0.97	4.55	0.79	4.34	4.02	1.00	4.58	0.83	4.31	3.97	1.03	4.40	0.83	5.63	4.75	1.30	5.88
3	10	1.83	9.29	8.38	2.20	9.67	1.62	8.29	7.06	1.92	8.85	1.82	8.79	7.78	2.16	9.19	1.95	9.31	7.88	2.25	10.24
3	11	1.74	7.14	6.51	2.00	7.14	1.72	7.63	6.67	1.99	7.65	1.73	7.42	6.69	2.00	7.55	1.70	7.11	6.54	1.98	7.37
3	12	2.00	8.26	7.68	2.28	8.63	1.84	7.83	7.25	2.11	7.86	1.89	7.85	7.42	2.18	8.23	1.90	8.10	7.50	2.18	8.37

obtained by calculating the weighted arithmetic mean according to Eq. (17). The first index, WA_t, weights the $\sqrt{\lambda_{ms}/\lambda_t}$ ratio using the explanation percentage of the eigenvalues extracted from total variation matrix ($\hat{\boldsymbol{\Sigma}}_t$). The second index, WA_{ms}, weights the $\sqrt{\lambda_{ms}/\lambda_t}$ ratio through the explanation percentage of the eigenvalues extracted from measurement system matrix ($\hat{\boldsymbol{\Sigma}}_{ms}$). On the other hand, the WG_t and WG_{ms} indexes are calculated using weighted geometric mean in Eq. (18). The first index, WG_t, weights the $\sqrt{\lambda_{ms}/\lambda_t}$ ratio using the explanation percentage of the eigenvalues extracted from total variation matrix ($\hat{\boldsymbol{\Sigma}}_t$). The second index, WG_{ms}, weights the $\sqrt{\lambda_{ms}/\lambda_t}$ ratio through the explanation percentage of the eigenvalues extracted from measurement system matrix ($\hat{\boldsymbol{\Sigma}}_{ms}$). The acceptance criteria of the multivariate measurement system are the same as described in Section 2.

4. Experimental setup

When structural integrity is not the most important requirement of a product (e.g., appliances, components to pumps, plugs, and connections), manufacturers have used mild steels. In producing such goods, the manufacturing process includes such features as satisfactory ductility, strength, and thermal treatment behavior as well as good machining conditions and excellent chip formation [33]. Researchers have recently given attention to the effect of noise factors in surface finishing for turning processes [34]. The quality of the surface finishing of a work piece can be assessed through roughness parameters such as: R_a (arithmetic average), R_y (maximum), R_z (ten point height), R_q (root mean square), and R_t (maximum peak to valley). For this empirical study, the turning process with five critical-to-quality characteristics consists of a multivariate GR&R study. The study investigates whether the variation of the roughness checker and the measurement procedure is significantly smaller than the process variation provided by the noise conditions (slenderness of the part, measuring position, and tool wear). The slenderness (S) relates the diameter (D) to the length (L) of the part, according to the relation $S = L/D$. The parts were classified as slender and non-slender, for the same part length, with $D = 30$ mm and $D = 50$ mm, respectively. The study adopted the following regions of measurement: close to the main spindle, the center, and close to the barrel. The tool wear noise considered a new tool and a worn tool, which was measured on the edge of approximately *0.3* mm [7].

The experiments were set up as follows. The work pieces, AISI 12L14 (0.09% C, 0.03% Si, 1.24% Mn, 0.046% P, 0.273% S, 0.15% Cr, 0.08% Ni, 0.26% Cu, 0.001% Al, 0.02% Mo, 0.28% Pb, 0.0079% N2), were machined on a NARDINI CNC lathe (see Fig. 2(a)) with 7.5 cv power and maximum rotation of 4000 rpm. The machining parameters used in this study were a cutting speed of 345 m min^{-1}, a feed rate of 0.086 mm rev^{-1}, and a depth of cut of 0.680 mm. Carbide inserts were used of ISO P35 class, coated with three toppings [Ti(C, N), Al_2O_3, TiN], (GC Sandvik 4035) geometry ISO SNMG 09 03 04 – PM, and tool holder ISO DSBNL 1616H 09. Using the experimental design and considering the aforementioned noise conditions, the GR&R study adopted $p = 12$ parts, $o = 3$ operators and $r = 4$ replicates. Table 1 presents the 720 measurements from the GR&R study (experiments were conducted randomly). Fig. 2(b) shows the equipment evaluated in this study—a portable roughness checker set to a cut-off length of 0.25 mm.

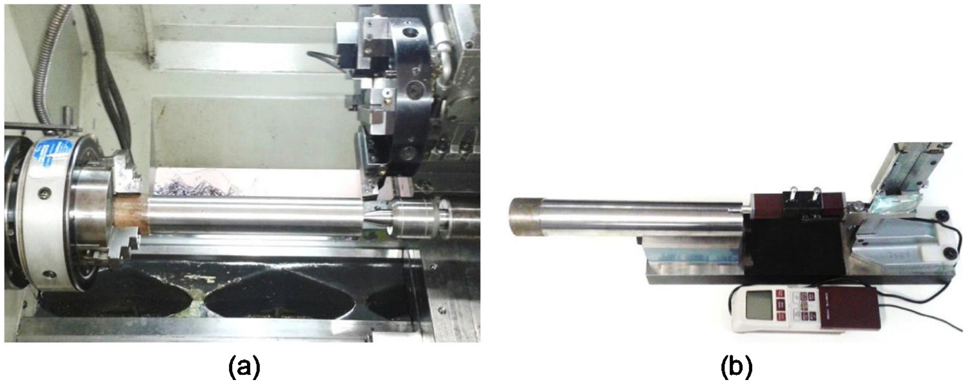

Fig. 2. (a) Mild steel machined on a NARDINI CNC lathe with conventional roller bearings and (b) Mitutoyo portable roughness checker model Surftest SJ-201P.

Table 2
Measurement system classification through the univariate method.

Source	R_a	R_y	R_z	R_q	R_t
σ_p	0.444	1.564	1.383	0.456	1.696
σ_{ms}	0.082	0.646	0.428	0.111	0.643
σ_t	0.452	1.693	1.448	0.470	1.813
%R&R	18.22	38.18	29.52	23.66	35.47

5. Result analysis

Initially, the roughness measurements in Table 1 were analyzed using a univariate approach according to the ANOVA model in Eq. (2). Variation components for process, measurement system, and total were computed and stored as standard deviations in Table 2. R_y and R_t classified the measurement system as unacceptable, whilst R_a, R_z and R_q as marginal. Assessing the entire data set in Table 1, the correlation structure among the roughness parameters was deemed significant, multivariate analysis should be performed so (see Table 3).

The data set in Table 1 was standardized using $(Y_i - \bar{Y})/\sigma_Y \; \forall i = 1, \ldots, n$ to minimize the effect of having variables in different scales for the multivariate analysis. This pretreatment determines that all roughness parameters are considered equally important [35]. The interaction effect was not significant for 0.05 of significance level; consequently, a two-way MANOVA model without interaction was adjusted to the multivariate data set, according to Eq. (12). First, mean square matrices were calculated using Eqs. (3), (4) and (13). These results observed in Eqs. (19)–(21) were then applied to obtain variance-covariance matrices for process ($\boldsymbol{\Sigma}_p$), measurement system ($\boldsymbol{\Sigma}_{ms}$) and total variation ($\boldsymbol{\Sigma}_t$) using the Eqs. (10), (11) and (14).

Table 3
Roughness parameters' correlations.

	R_a	R_y	R_z	R_q	R_t
R_a	1.000				
	–				
R_y	0.621[a]	1.000			
	0.000[b]	–			
R_z	0.819	0.906	1.000		
	0.000	0.000	–		
R_q	0.985	0.701	0.891	1.000	
	0.000	0.000	0.000	–	
R_t	0.601	0.989	0.892	0.681	1.000
	0.000	0.000	0.000	0.000	–

[a] Pearson correlation
[b] p-value.

Compositions of the mean square and variance-covariance matrices follow the same structure of the correlation matrix in Table 3:

$$\mathbf{MSP} = \begin{pmatrix} 12.557 & 8.139 & 10.401 & 12.397 & 7.756 \\ 8.139 & 11.803 & 11.239 & 9.106 & 11.596 \\ 10.401 & 11.239 & 12.500 & 11.165 & 10.953 \\ 12.397 & 9.106 & 11.165 & 12.500 & 8.731 \\ 7.756 & 11.596 & 10.953 & 8.731 & 11.517 \end{pmatrix} \quad (19)$$

$$\mathbf{MSO} = \begin{pmatrix} 0.002 & -0.004 & 0.003 & 0.002 & 0.006 \\ -0.004 & 0.011 & -0.004 & -0.004 & -0.014 \\ 0.003 & -0.004 & 0.003 & 0.003 & 0.006 \\ 0.002 & -0.004 & 0.003 & 0.002 & 0.006 \\ 0.006 & -0.014 & 0.006 & 0.006 & 0.018 \end{pmatrix} \quad (20)$$

$$\mathbf{MSE} = \begin{pmatrix} 0.037 & 0.028 & 0.043 & 0.039 & 0.029 \\ 0.028 & 0.101 & 0.061 & 0.037 & 0.105 \\ 0.043 & 0.061 & 0.081 & 0.051 & 0.072 \\ 0.039 & 0.037 & 0.051 & 0.042 & 0.040 \\ 0.029 & 0.105 & 0.072 & 0.040 & 0.125 \end{pmatrix} \quad (21)$$

Eigenvalues extracted from $\boldsymbol{\Sigma}_p$, $\boldsymbol{\Sigma}_{ms}$, and $\boldsymbol{\Sigma}_t$ matrices, in Table 4, are required to estimate the multivariate evaluation index for assessing this measurement system. Using Eq. (16) and eigenvalues in Table 4, G = 44.64% classified the measurement system as unacceptable. Nevertheless, when compared to the univariate %R&R indexes, G = 44.64% seems to misestimate the multivariate index for assessing the measurement system. Therefore, four multivariate indexes were proposed in addition to the G index. A simple geometric mean does not determine greater importance to the most significant pair of eigenvalues, extracted from variance–covariance matrices. Thus, WA_t, WA_{ms}, WG_t and WG_{ms} were calculated using weighted approaches upon $\sqrt{\lambda_{ms}/\lambda_t}$ ratio. According to Eqs. (17) and (18), the new multivariate indexes are shown in Eqs. (22)–(25):

$$WA_t = 29.8 \times 0.845 + 23.7 \times 0.137 + 39.9 \times 0.015 + 88.8 \times 0.002 + 70.7 \times 0.001 = 29.30\% \quad (22)$$

$$WA_{ms} = 29.8 \times 0.860 + 23.7 \times 0.088 + 39.9 \times 0.027 + 88.8 \times 0.019 + 70.7 \times 0.006 = 30.92\% \quad (23)$$

$$WG_t = 29.8^{0.845} \times 23.7^{0.137} \times 39.9^{0.015} \times 88.8^{0.002} \times 70.7^{0.001} = 29.12\% \quad (24)$$

Table 4
Eigenvalues and W_i of the variance-covariance matrices $\hat{\Sigma}_p$, $\hat{\mathbf{\Sigma}}_{ms}$ and $\hat{\Sigma}_t$.

Sources of variation		Eigenvalues				
		λ_1 (W_1[b])	λ_2 (W_2)	λ_3 (W_3)	λ_4 (W_4)	λ_5 (W_5)
Process	$\hat{\Sigma}_p = \begin{pmatrix} 1.043^{a} & 0.676 & 0.863 & 1.030 & 0.644 \\ 0.676 & 0.975 & 0.932 & 0.756 & 0.958 \\ 0.863 & 0.932 & 1.035 & 0.926 & 0.907 \\ 1.030 & 0.756 & 0.926 & 1.038 & 0.724 \\ 0.644 & 0.958 & 0.907 & 0.724 & 0.949 \end{pmatrix}$	4.195 (85.0)	0.673 (13.6)	0.063 (1.3)	0.002 (0.0)	0.000 (0.0)
Measurement system	$\hat{\Sigma}_{ms} = \begin{pmatrix} 0.037 & 0.027 & 0.042 & 0.038 & 0.028 \\ 0.027 & 0.099 & 0.059 & 0.036 & 0.103 \\ 0.042 & 0.059 & 0.079 & 0.050 & 0.070 \\ 0.038 & 0.036 & 0.050 & 0.041 & 0.039 \\ 0.028 & 0.103 & 0.070 & 0.039 & 0.123 \end{pmatrix}$	0.406 (86.0)	0.042 (8.8)	0.013 (2.7)	0.009 (1.9)	0.003 (0.6)
Total variation	$\hat{\Sigma}_t = \begin{pmatrix} 1.080 & 0.703 & 0.905 & 1.068 & 0.672 \\ 0.703 & 1.074 & 0.991 & 0.792 & 1.061 \\ 0.905 & 0.991 & 1.114 & 0.977 & 0.977 \\ 1.068 & 0.792 & 0.977 & 1.080 & 0.764 \\ 0.672 & 1.061 & 0.977 & 0.764 & 1.072 \end{pmatrix}$	4.567 (84.5)	0.743 (13.7)	0.079 (1.5)	0.011 (0.2)	0.006 (0.1)

[a] Standardized data.
[b] $W_i = (\lambda_i / \sum_{j=1}^{q} \lambda_j) \times 100\% \quad i = 1, 2, \ldots, q.$

$$WG_{ms} = 29.8^{0.860} \times 23.7^{0.088} \times 39.9^{0.027} \times 88.8^{0.019} \times 70.7^{0.006} = 30.23\% \quad (25)$$

To compare G index to multivariate indexes proposed in this paper, the 95% confidence interval was computed using Eq. (26) [7]:

$$CI = \bar{Y} \pm t_{N-1,\alpha/2} \frac{s}{\sqrt{N}} \quad (26)$$

In this expression, $\bar{Y}$ is the mean of $\%R\&R$ between $\mathbf{Y}_1$, $\mathbf{Y}_2$, $\mathbf{Y}_3$, and $\mathbf{Y}_4$; s is the standard deviation; N is the sample size and $t_{N-1,\alpha}$ is the $(1-\alpha)$100th percentile of a t distribution with $(N-1)$ degrees of freedom.

When compared to the 95% confidence interval [18.797; 39.23] obtained by univariate indexes, G index seems to misestimate the multivariate evaluation index. Additional indexes proposed in this article seem to obtain more coherent results in assessing multivariate GR&R studies, since all indexes were calculated inside the 95% confidence interval. Due to these findings, additional numerical examples taken from the literature were considered to examine more scenarios involving measurement systems and correlation structures among Ys.

6. Additional numerical examples

6.1. Automotive body stamped-panel measurement system

Majeske [3] presented an industrial application in which an automobile manufacturer constructed a hard gauge to measure four Ys on a sheet-metal body panel. His study consisted of a GR&R study with $p=5$ parts, $o=2$ operators, and $r=3$ replicates. The critical-to-quality characteristics "M" in his paper were called here Y. Adjusting the data to an ANOVA model, the $\%R\&R$ indexes for the measurement system were: $\%R\&R_{Y1} = 22.20\%$, $\%R\&R_{Y2} = 15.66\%$, $\%R\&R_{Y3} = 15.09\%$ and $\%R\&R_{Y4} = 9.26\%$. Now, considering the correlation structure among the Ys, a single index that represented the multivariate measurement system was calculated. The result was $G = 12.28\%$. The same criterion in Eq. (26) was applied to assess the indexes' adequacy. The results for G, WA_t, WA_{ms}, WG_t and WG_{ms} were estimated for both turning process (Section 5) and automotive body stamped-panel (Section 6.1) and are presented in Fig. 3.

As can be seen from Fig. 3, no significant differences were found among the multivariate indexes for the automobile manufacturer data. All indexes were calculated inside the confidence interval [7.13%; 23.97%]. However, turning back to the experimental evidence in the turning process, a motivation lingered to investigate why G index was estimated far from the confidence interval.

6.2. A simulation study

A recent work by Peruchi et al. [7] provided a simulation study for a multivariate GR&R study using the same setup as that used in Majeske [3]. In their article, 15 scenarios were simulated considering both several correlation structures for Ys and different types of measurement systems. Three types of measurement systems were considered: acceptable (AC: $\%R\&R < 10\%$), marginal (MA: $10\% < \%R\&R < 30\%$), and unacceptable (UN: $\%R\&R > 30\%$). The correlation structure were classified as: very low (VL: $W_1 \le 65\%$), low (L: $65\% < W_1 \le 75\%$), medium (M: $75\% < W_1 \le 85\%$), high (H: $85\% < W_1 \le 95\%$), and very high (VH: $W_1 > 95\%$). W_1 is the result obtained from $\lambda_{T_1} / \sum_{j=1}^{q} \lambda_{T_j}$. The criterion to assess indexes' adequacy was the same as in Eq. (26). Table 5 shows their results and the estimates of the four weighted approaches are also added.

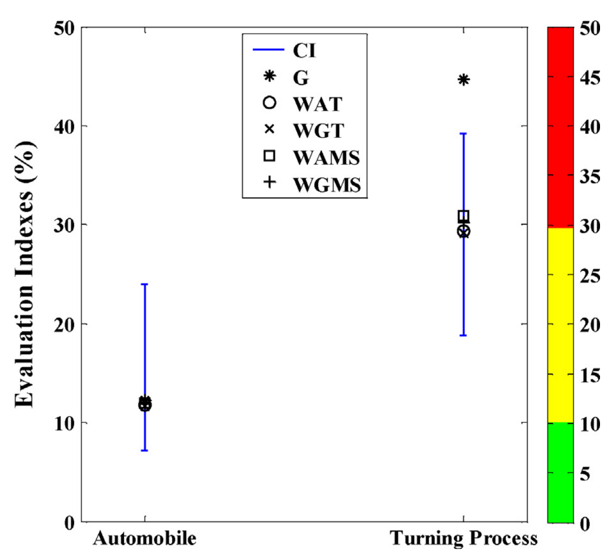

Fig. 3. Multivariate evaluation indexes for automobile manufacturer and turning process applications.

Table 5
Results for calculations of univariate index, 95% confidence interval and multivariate indexes.

Scenario			Univariate (%R&R)				Mean CI			Multivariate				
S	MS	Corr.	Y_1	Y_2	Y_3	Y_4	$\bar{Y}$	LCL	UCL	G	WA_t	WG_t	WA_{ms}	WG_{ms}
S1		VL	49.9	39.3	38.3	34.1	40.42	29.69	51.14	10.78	31.62	18.28	48.84	48.51
S2		L	42.2	55.5	44.3	39.8	45.44	34.42	56.47	13.30	36.50	30.28	45.42	44.94
S3	UN	M	40.8	52.4	42.6	36.9	43.18	32.63	53.72	11.32	38.27	31.45	45.76	45.51
S4		H	45.3	33.2	41.2	47.8	41.86	31.70	52.03	28.15	42.95	42.20	44.33	44.14
S5		VH	31.1	34.9	37.8	41.1	36.21	29.45	42.97	64.09	35.81	35.79	36.05	35.94
S6		VL	15.8	14.1	13.7	10.2	13.48	9.75	17.21	4.97	9.62	6.48	15.67	15.38
S7		L	18.6	27.2	21.3	24.1	22.82	16.95	28.69	10.04	19.86	14.68	27.15	26.87
S8	MA	M	15.5	23.7	17.0	14.6	17.69	11.16	24.21	5.40	15.38	13.24	18.02	17.90
S9		H	13.2	10.3	13.6	16.9	13.50	9.19	17.80	14.31	14.37	14.35	14.44	14.43
S10		VH	15.2	19.0	19.7	20.9	18.70	14.80	22.59	47.23	16.95	16.94	17.33	17.12
S11		VL	8.4	6.3	4.9	5.3	6.22	3.67	8.77	4.08	5.00	4.41	6.75	6.49
S12		L	5.6	4.6	6.7	5.4	5.54	4.15	6.92	2.01	4.70	3.53	6.18	6.13
S13	AC	M	6.2	9.6	6.6	5.9	7.07	4.37	9.76	2.28	6.07	5.32	7.05	7.00
S14		H	5.7	4.5	5.9	7.3	5.84	4.00	7.69	7.22	6.58	6.56	6.65	6.63
S15		VH	6.5	7.6	8.6	9.2	7.95	6.07	9.83	39.35	7.78	7.78	8.18	7.89

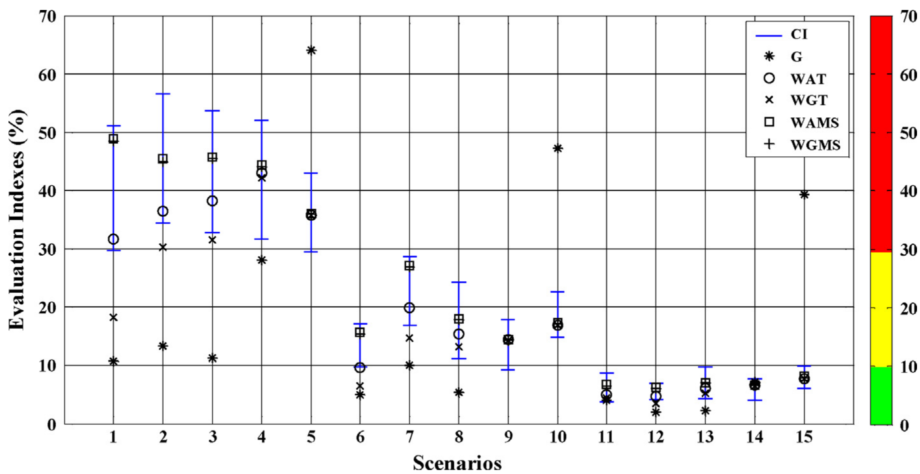

Fig. 4. Multivariate evaluation indexes estimated for 15 distinct scenarios.

Moreover, Fig. 4 graphically presents how accurate the multivariate indexes were estimated compared to the 95% confidence interval. In the same manner as Peruchi et al. [7], this study will lead the analysis and comparison in two ways: intra- and inter-indexes. From a general perspective, the intra-index analysis will show an overview of the indexes' performance; the inter-index analysis, in contrast, will present a more detailed assessment addressing the indexes' deviations from the confidence intervals.

The intra-index analysis verified that the WA_{ms} and WG_{ms} indexes were more accurate than G, WA_t, and WG_t. Only in scenarios S9, S11, and S14 was G index estimated within the confidence interval. WA_t and WG_t indexes failed in one and six scenarios, respectively. As can be seen in Fig. 4, WA_{ms} and WG_{ms} indexes were estimated within the confidence interval in all 15 scenarios evaluated.

For the inter-index analysis, G index was verified to be calculated within the confidence interval in only S9, S11, and S14 scenarios. This index was obtained using geometric mean of the $\sqrt{\lambda_{ms}/\lambda_t}$ ratio according to the amount of quality characteristics. Geometric mean provides the same degree of importance in the analysis of each pair of eigenvalues. As Peruchi et al. [7] have already proved that G index may not represent well the performance of the measurement system if the individual ratio $\sqrt{\lambda_{ms}/\lambda_t}$ for each pair of eigenvalues, extracted from $\hat{\boldsymbol{\Sigma}}_{ms}$ and $\hat{\boldsymbol{\Sigma}}_t$, provide different interpretations. It is essential to emphasize that the first eigenvalues have a greater percentage of explaining the measured phenomenon than do the last. Therefore, weighted approaches in multivariate analysis of variance for GR&R studies were proposed to improve G index estimation.

In the inter-index analysis by WA_t and WG_t, the weighted approach through the explanation percentage of the eigenvalues extracted from $\hat{\boldsymbol{\Sigma}}_t$: $W_i = (\lambda_{t_i}/\sum_{j=1}^{q}\lambda_{t_j})$ was unsatisfactory. These indexes (mainly WG_t) failed in scenarios with correlations deemed lower due to higher weights assigned to less significant $\sqrt{\lambda_{ms}/\lambda_t}$ ratios.

In the inter-index analysis by WA_{ms} and WG_{ms}, the explanation percentage of the eigenvalues extracted from $\hat{\boldsymbol{\Sigma}}_{ms}$: $W_i = (\lambda_{ms_i}/\sum_{j=1}^{q}\lambda_{ms_j})$ proved to be the most effective weighted approach for assessing a multivariate measurement system. In most scenarios, the first pair of eigenvalues to calculate $\sqrt{\lambda_{ms}/\lambda_t}$ ratio received a greater degree of importance, including scenarios with a lower correlation structure. Conceptually, the weighted approach using the explanation percentages of the eigenvalues extracted from the measurement system matrix make more sense than from the total variation matrix, in estimating the evaluation index for GR&R studies.

7. Conclusions

This article has investigated the multivariate analysis of variance as applied to GR&R studies. In assessing measurement systems with multiple correlated characteristics, the MANOVA method has demonstrated some drawbacks when estimating the multivariate evaluation index. The main contribution of this research has been to propose new multivariate indexes based on weighted approaches to overcome such drawbacks. The empirical findings suggest that G index may misclassify the multivariate measurement system due to its significant shift from the univariate estimates. In this case, $G = 44.64\%$ was estimated outside the confidence interval [18.78%; 39.23%]. Analyzing the first literature data set, no significant differences were found between G indexes and the weighted indexes proposed in this study. All indexes were calculated inside the confidence interval [7.13%; 23.97%]. However, the second literature data set presented strong evidence of G index limitations and WA_{ms} and WG_{ms} as being the best weighted indexes. Taking into account the empirical and literature findings, WA_{ms} and WG_{ms} better estimated the multivariate evaluation index than did G, WA_t and WG_t. This is because the first pair of eigenvalues to calculate $\sqrt{\lambda_{ms}/\lambda_t}$ ratio received a greater degree of importance, including scenarios with lower correlations. Accordingly, the weighted approach using the variation in regard to the measurement system matrix was shown to be the best manner to estimate the evaluation index in multivariate GR&R studies with the MANOVA method.

Further research might explore the effectiveness of using weighted approaches to estimate other evaluation indexes (*ndc* and *P*/*T*, for example) in multivariate GR&R studies. In this work, only the MANOVA method was applied to assess measurement systems. A further study could also compare multivariate evaluation indexes obtained using MANOVA and PCA methods.

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Appendix A. Supplementary data

Supplementary material related to this article can be found, in the online version, at http://dx.doi.org/10.1016/j.precisioneng.2014.03.001.

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