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ORIGINAL ARTICLE

Impact of stochastic industrial variables on the cost optimization of AISI 52100 hardened-steel turning process

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Abstract

An optimization problem of the AISI 52100 hard-steel turning process is examined. A new approach is presented in which not only the machine parameters (cutting speed, feed rate, and depth of cut) but also the stochastic industrial variables of setup time, insert changing time, batch size, machine and labor costs, tool holder price, tool holder life, and insert price are considered. By representing each of these variables by a given probability distribution, the goal was to analyze their impact on the total process cost per piece (K_p). Experiments were carried out following a central composite design to model tool life (T), average surface roughness (R_a), and peak-to-valley surface roughness (R_t) using a response surface methodology. Then, stochastic programming was used to model K_p's expected value and standard deviation. The approach to the optimization problem aimed to maximize the probability for the cost to be less than a target value, subject to the experimental space and to maximum values of both R_a and R_t. The results were optimal values for the cutting conditions that provide a suitable confidence interval for K_p. The most-significant industrial variables on K_p were ranked. In addition, it was found that, in the addressed case, cutting conditions for maximum tool life actually increase K_p.

Keywords Stochastic programming · Hardened-steel turning · Process cost optimization

1 Introduction

Recently, advances in the machining of hardened steels, such as hard turning, have significantly contributed to product quality in manufacturing industries [1–3]. In fact, hard turning is a manufacturing process widely applied in industry. Compared with grinding, hard turning can provide equal or even better surface finish [4] with higher material removal rates [5]. Other benefits provided by hard turning include coolant reduction or elimination, process cost reduction, productivity increase, improved material properties, and reduced power consumption [6–8].

Gears, shafts, bearings, bushes, dies, crushing cones, and jet engine mounting are some of the applications of hardened steels [9]. In particular, AISI 52100 hardened steel is frequently used to manufacture bearings, ball screws, gauges, axles, and joints because of its strength and corrosion resistance [10]. AISI 52100 is considered to be one of the hard-to-cut steel alloys [11] in terms of cutting tool materials and economical machining.

Nevertheless, only a few studies on hardened-steel turning optimization have considered the impact of industrial variables and their effect on the variability of the process cost. Industrial variables include setup time, insert changing time, batch size, and others [12]. Most of them are stochastic, and some are not controllable. There are already different stochastic-programming models available in the literature [13], and some researchers have already applied them in manufacturing systems to analyze setup times [14], batch size [15], and machines and labor [16]. Within this context, this study aimed to optimize the total process cost per piece for AISI 52100 hardened-steel turning by also taking into account the following stochastic variables: setup time, insert changing time, batch size, machine and labor cost, tool holder price, tool holder life, and insert price.

This work is structured as follows. In Section 2, a review of the literature about response surface methodology (RSM), total process cost per piece in turning, and stochastic programming is presented. In Section 2.3, an equation used to calculate

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the variance of a continuous function dependent on stochastic variables is given. The materials and methods are described in Section 3, including the designed experiments and the mathematical modeling. Results and discussion are presented in Section 4. The equation was validated by a real case study and using Monte Carlo simulation. Tool life maximization and its impact on machining cost were also analyzed quantitatively. Finally, Section 5 presents the conclusions.

2 Literature review

2.1 Design of experiments and mathematical modeling

Many studies have already used the design of experiments (DOE) strategy to analyze different sorts of industrial process, such as resistance spot welding [17, 18], the 3D-printing process [19], laser beam machining [20], and hard turning [21]. In particular, research on hard turning has progressed in different directions over time, but a large number of studies have focused on mathematical modeling and optimization. In such approaches, DOE is frequently used, because it makes possible the analysis of how each of the decision variables and their interactions affect the results of interest [22]. DOE often reduces the number of experiments needed to analyze the process and to build analytical models of the results of interest, which decreases the experimental cost [23].

The analytical models are commonly built by the RSM. The RSM is a DOE method composed of statistical and mathematical techniques used to model an objective function dependent on multiple input variables [24]. The RSM is known as a practical and economical experimental method and has recently been used to model outputs of machining processes [20, 21, 25].

A response surface model can be represented by a second-order polynomial, as in Eq. (1).

$$Y=\beta_0+\sum_{i=1}^{k}\beta_i x_i+\sum_{i=1}^{k}\beta_{ii}x_1^2+\sum_{i<j}\sum\beta_{ij}x_i x_j+\varepsilon \quad (1)$$

For instance, a central composite design (CCD) [24] can be used to define the experimental runs. After the experiments are executed, a regression model, such as the ordinary least-squares (OLS) method, is used to build the mathematical model. If the model presents satisfactory adjustments and residuals, then it is possible to formulate and solve an optimization problem.

In hardened-steel turning optimization, surface roughness, tool life, and other outputs are generally represented by response surface models. Some other process outputs, however, can be directly calculated by equations available in the literature and, for this reason, do not necessarily require the use of the RSM. One example is the process cost, which is described in Section 2.2.

2.2 Total process cost per piece in turning

The total cost of machining a piece is considered to be one of the most important aspects in metal cutting industries for manufacturing a product [1]. It comprises manufacturing costs, which are directly related to the process (such as machines, labor, and tools), and other indirect costs (quality control, raw materials, indirect labor, etc.) [26]. The manufacturing cost of a piece is also defined as the sum of operation, tool, and tool change costs per piece [21]. Diniz, Marcondes e Copini [12] provided Eq. (2), which is used to calculate the manufacturing cost, also known as the total process cost per piece (K_p).

$$K_p=T_t\frac{(S_h+S_m)}{60}+\frac{C_t}{T}\left(\frac{K_{th}}{N_{th}}+\frac{K_i}{N_i}\right) \quad (2)$$

In Eq. (2), T_t is the total cycle time, measured in minutes; S_m+S_h are, respectively, labor and machine costs per hour; C_t is the cutting time (min); T refers to tool life (min); K_{th} is the tool holder cost; N_{th} is the average tool holder life, measured in edges; K_i is the insert cost; and N_i is the number of cutting edges of the insert. T_t is calculated by Eq. (3).

$$T_t=C_t+t_s+t_a+\frac{t_p}{Z}+\frac{N_t}{Z}t_i$$
$$T_t=C_t+t_s+\frac{t_p}{Z}+\frac{N_t}{Z}\text{ti} \quad (3)$$

where t_s is the secondary time, t_a is the tool approximation and retreat time, t_p is the setup time, Z is the batch size, N_t is the number of tool changes in the same batch, and t_i is the insert changing time. C_t is calculated by Eq. (4).

$$C_t=\frac{l_f\times\pi\times D}{1000\times f\times V_c} \quad (4)$$

where l_f is the piece length, d is the piece diameter, f is the feed rate, and V_c is the cutting speed. Equation (5) is used to calculate the number of tool changes (N_t).

$$N_t=Max.\left[0,I\left(Z\frac{C_t}{T}-1\right)\right] \quad (5)$$

where $I\left(Z\frac{C_t}{T}-1\right)$ is the smallest integer number greater than or equal to $Z\frac{C_t}{T}-1$.

Based on Eqs. (2), (3), (4), and (5), there are many variables that influence the total process cost per piece (K_p). Among them, cutting speed (V_c) and feed rate (f) are quantitative cutting conditions. Along with depth of cut (d), these three machine parameters have been used as decision variables in several works on hard-turning process optimization [4, 10, 27, 28]. The most common results of interest are average surface roughness, material removal rate, cutting forces, tool wear, process cost, and energy consumption. When it comes to the manufacturing industry, the optimization of results related to cost, productivity, and quality is frequently the main goal [29].

Some industrial variables are stochastic and uncontrollable, which means that production systems must be aware of how the variability affects their results. Among the variables related to the calculation of K_p, the following ones have already been pointed out and investigated in previous studies.

- Setup time (t_p) and insert changing time (t_i): Samaddar [14] presented some findings on how the setup time (t_p) variance may affect a production system. In fact, it is possible that setup times are stochastic in practice, so the solutions of deterministic models may result in deteriorated quality if applied to real-life problems, according to Tas et al. [30]. The authors also stated that there is always inherent variability in the execution time of a specific activity. Because insert changes are setup activities, the insert changing time (t_i) should also be considered a stochastic variable.
- Batch size (Z), labor, and machine costs ($S_m + S_h$): An increasing number of companies have been adopting a just-in-time philosophy [31]. As a result, batch sizes (Z) may vary according to the customers' demands, which are usually considered random [32]. In addition, Francas et al. [16] investigated how machine and labor flexibility may reduce production costs of manufacturing networks. In these scenarios, machine and labor costs ($S_m + S_h$) also have a stochastic nature.
- Tool holder price (K_{th}), insert price (K_i), and tool holder life (N_{th}): Canyakmaz et al. [33] studied the impact of stochastic item prices on the optimal inventory setting. According to the authors, price uncertainties are one of the most critical challenges of manufacturers, and such uncertainties may be caused by unstable economies, strikes, exchange rates, and other contributing factors. Hence, tool holder price (K_{th}) and insert price (K_i) may be included in the group of stochastic variables related to K_p. Finally, in real-life operations, predicting the tool holder life (N_{th}) is extremely difficult, but it may also be approximated to a given probability distribution.

Therefore, some of the industrial variables related to K_p could be included in the optimization problem using stochastic-programming methods.

2.3 Stochastic programming

2.3.1 Variance of a continuous function of normal variables

There are already different strategies to model the variance of analytical models $y = \mathbf{z}^{\mathrm{T}}\boldsymbol{\beta}$ [13, 25], which are commonly used in the RSM. However, it is also possible to model the variance of a general continuous function dependent on stochastic variables.

Let $f(\mathbf{x})$ be a continuous function dependent on vector $\mathbf{x} = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$ composed by two normally distributed variables. Considering a Taylor series limited to the linear term and applied for a, one obtains Eq. (6)

$$f(\mathbf{X}) = f(x_1, x_2) = f(\mu_{x1}, \mu_{x2}) = + \sum_{i=1}^{n=2} (x_1 - \mu_i) \frac{\partial f(x_1, x_2)}{\partial x_i} \tag{6}$$

or Eq. (7)

$$f(x_1, x_2) - f(\mu_{x_1}, \mu_{x_2}) = (x_1 - \mu_{x_1}) \frac{\partial f(x_1, x_2)}{\partial x_1} + (x_2 - \mu_{x_2}) \frac{\partial f(x_1, x_2)}{\partial x_2} \tag{7}$$

Taking both sides to the second order and applying the expected value, one obtains

$$E[f(x_1, x_2) - f(\mu_{x_1}, \mu_{x_2})]^2 = E\left[(x_1 - \mu_{x_1}) \frac{\partial f(x_1, x_2)}{\partial x_1} + (x_2 - \mu_{x_2}) \frac{\partial f(x_1, x_2)}{\partial x_2}\right]^2 \tag{8}$$

or

$$Var[f(x_1, x_2)] = E\left\{\left[(x_1 - \mu_{x_1}) \frac{\partial f(x_1, x_2)}{\partial x_1}\right]^2 + \left[(x_2 - \mu_{x_2}) \frac{\partial f(x_1, x_2)}{\partial x_2}\right]^2\right\} + E\left\{2 \times \left[(x_1 - \mu_{x_1})(x_2 - \mu_{x_2}) \frac{\partial f(x_1, x_2)}{\partial x_1} \frac{\partial f(x_1, x_2)}{\partial x_2}\right]^2\right\} \tag{9}$$

Hence, the variance of a function of two independent variables evaluated in the mean vector $\boldsymbol{\mu}$ becomes

$$Var[f(\mathbf{x})] = \left[\frac{\partial f(\boldsymbol{\mu})}{\partial x_1}\right]^2 \sigma_{x_1}^2 + \left[\frac{\partial f(\boldsymbol{\mu})}{\partial x_2}\right]^2 \sigma_{x_2}^2 + 2 \times \left[\frac{\partial f(\boldsymbol{\mu})}{\partial x_1}\right]\left[\frac{\partial f(\boldsymbol{\mu})}{\partial x_2}\right] \sigma_{x_1}\sigma_{x_2} \tag{10}$$

Likewise, the variance of a function dependent on n variables is

$$Var[f(\mathbf{x})] = \sum_{i=1}^{n} \left[\frac{\partial f(\boldsymbol{\mu})}{\partial x_i}\right]^2 \sigma_{x_i}^2 + 2 \times \sum_{i=1}^{n-1} \sum_{j=i+1}^{n} \left[\frac{\partial f(\boldsymbol{\mu})}{\partial x_i}\right] \left[\frac{\partial f(\boldsymbol{\mu})}{\partial x_j}\right] \sigma_{x_i}\sigma_{x_j} \quad (11)$$

In the format of matrices, it is possible to use Eq. (11) to obtain the standard deviation (SD) of $f(\mathbf{x})$ as in Eq. (12)

$$SD[f(\mathbf{x})] = \sqrt{Var[f(\mathbf{x})]} = \sqrt{\nabla f(\mathbf{x})^T \Sigma \nabla f(\mathbf{x})} \quad (12)$$

where $\nabla f(\mathbf{x})$ is the gradient vector of $f(\boldsymbol{x})$, and $\boldsymbol{\Sigma}$ is the variance and covariance matrix of the variables in $\mathbf{x}$, as in Eq. (13).

$$\boldsymbol{\Sigma} = \begin{bmatrix} \sigma_{x_1}^2 & \cdots & \sigma_{x_n x_1} \\ \vdots & \ddots & \vdots \\ \sigma_{x_1 x_n} & \cdots & \sigma_{x_n}^2 \end{bmatrix} \quad (13)$$

2.3.2 Modeling the probability of attending a maximum cost

Equation (14) presents an optimization problem whose goal is to maximize the probability of the objective function $f(\boldsymbol{x})$ being less than or equal to its upper specification limit (USL) submitted to the problem constraints $g_i(\mathbf{x})$.

$$\begin{cases} \max_x P[f(\mathbf{x}) \le USL] = \int_{-\infty}^{USL} \phi\left\{E[f(\mathbf{x})], \quad \sqrt{Var[f(\mathbf{x})]}\right\} \\ s.t.: \\ g_i(\mathbf{x}) \le u_i; \quad \forall i = 1, \ldots, k \end{cases} \quad (14)$$

where $\varphi\{\}$ is the probability density function composed by the expected value and the standard deviation of $f(\mathbf{x})$ [13].

For instance, if $f(\mathbf{x})$ is the mathematical model for the process cost, then the goal is to maximize the probability of the cost being less than or equal to a predefined value USL. The constraints $g_i(\mathbf{x})$ may include the experimental space constraint or even other output models that are not defined as objective functions, but as other constraints of the problem: surface roughness, material removal rate, and others.

3 Materials and methods

3.1 Experimental procedure

Experiments were executed in a CNC Nardini Logic 175 lathe (Fig. 1a), with a maximum rotation speed of 4000 rpm and a cutting power of 5.5 kW. The workpieces of AISI 52100 had the following chemical composition: 1.03% C, 0.23% Si, 0.35% Mn, 1.40% Cr, 0.04% Mo, 0.11% Ni, 0.001% S, and 0.01% P. Their initial dimensions were Ø 49 × 50 mm, and they were quenched and tempered, providing a hardness between 49 and 52 HRC up to a depth of 3 mm below the surface. The hard-turning process was executed with wiper mixed ceramic (Al_2O_3 + TiC) inserts (CNGA 120408 S01525WH), coated with a thin layer of titanium nitride (TiN). The tool holder had a negative geometry with ISO code DCLNL 1616H12 and an entering angle χr = 95°.

To measure tool life, wiper inserts were worn until their flank wear (VB_C) indicator on the tool tip reached 0.30 mm. This was the adopted criterion for the end of tool life, and it was measured by an optical microscope. The arithmetic mean roughness (R_a) and the maximum peak-to-valley roughness (R_t), both in micrometers, were measured at the end of life of each wiper insert. These responses were measured using a portable roughmeter set to a cutoff length of 0.8 mm. The measurements were taken at three different points of the workpiece, as indicated in Fig. 1b. Each point was measured four times, and their mean value was considered. More details of the method are described elsewhere [34].

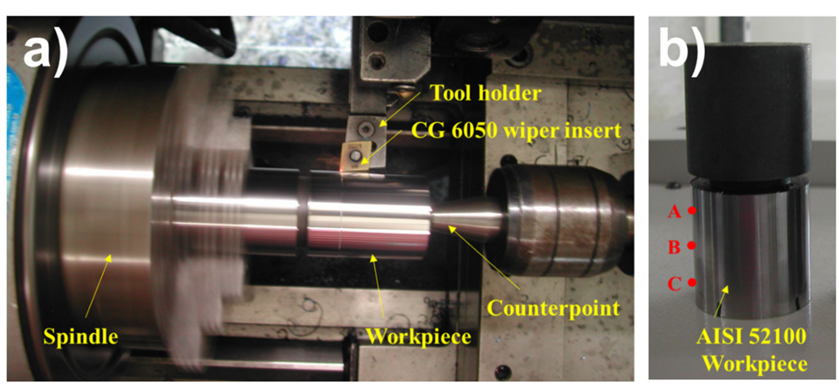

Fig. 1 **a** AISI 52100 hard-turning process with wiper inserts and **b** surface roughness measurement positions

3.2 Mathematical models and optimization problem

The RSM was applied to build analytical models for tool life (T), average surface roughness (R_a), and maximum peak-to-valley surface roughness (R_t). Using the CCD, 19 experiments were carried out: eight factorial points, six axial points, and five center points. The decision variables were cutting speed (V_c), feed rate (f), and depth of cut (d). Table 1 presents the decision variables, their units, and levels, both with coded and decoded values.

The distance from the axial points to the center point was calculated by $\rho = \sqrt[4]{k^n}$, where k is the number of factorial levels, and n is the number of decision variables. Table 2 shows the experimental data and results. After the experiments, the OLS method was used to build analytical second-order polynomial models for T, R_a, and R_t using Eq. (1), and their coefficients are presented in Table 3. The models' adjustments were considered adequate, because their R^2_{adj} values were higher than 89%.

The total process cost per piece (K_p), total cycle time (T_t), and cutting time (C_t) were calculated using Eqs. (2), (3), and (4), respectively, and the values of the industrial variables are presented in Table 4. The material removal rate (MRR) was obtained by simply multiplying cutting speed by feed rate by depth of cut.

As justified in Section 2.2, out of the 12 industrial variables presented in Table 4, only seven were modeled as stochastic: setup time (t_p), insert changing time (t_i), batch size (Z), machine and labor costs ($S_h + S_m$), tool holder price (K_{th}), average tool holder life (N_{th}), and insert price (K_i).

The batch size is the only discrete variable and follows a Poisson distribution, which can be approximated to a normal distribution using $Z = \frac{X-\lambda}{\sqrt{\lambda}}$ [35]. For the other six variables, which are continuous ones, values were generated for each variable using different continuous probability distributions. However, the analysis refers to a period of time, so the mean values of the variables were calculated for every week during 3 months of simulation, and the expected values were computed. The samples composed by the expected values follow a normal distribution, according to the central limit theorem [36]. For this reason, normal distributions were used to represent the stochastic variables. The relative standard deviation was 10%. Because this is a theoretical analysis, there was no evidence to infer that the stochastic variables were significantly correlated in the present

Table 1 Levels of the decision variables

Decision variables	Levels				
Coded levels	− 1.682	− 1.000	0.000	1.000	1.682
V_c (m/min)	186.4	200.0	220.0	240.0	253.6
f (mm/v)	0.132	0.200	0.300	0.400	0.468
d (mm)	0.100	0.150	0.225	0.300	0.351

Source: Campos et al. [34]

Table 2 Experimental data and results

Run	Decision variables			Results of interest						
	V_c	f	d	T	R_a	R_t	K_p	T_t	C_t	MRR
1	200	0.20	0.15	17.21	0.25	1.41	0.76	0.86	0.19	6.00
2	240	0.20	0.15	11.37	0.27	1.72	0.76	0.83	0.16	7.20
3	200	0.40	0.15	5.96	0.31	2.12	0.72	0.77	0.10	12.00
4	240	0.40	0.15	4.48	0.30	2.15	0.72	0.76	0.08	14.40
5	200	0.20	0.30	9.42	0.25	1.45	0.84	0.87	0.19	12.00
6	240	0.20	0.30	7.37	0.25	1.58	0.82	0.84	0.16	14.40
7	200	0.40	0.30	4.03	0.34	2.01	0.79	0.78	0.10	24.00
8	240	0.40	0.30	6.10	0.29	1.99	0.68	0.75	0.08	28.80
9	186	0.30	0.22	9.51	0.29	1.69	0.74	0.81	0.14	12.28
10	254	0.30	0.22	6.86	0.26	1.81	0.71	0.77	0.10	16.76
11	220	0.13	0.22	14.18	0.21	1.54	0.89	0.95	0.27	6.29
12	220	0.47	0.22	4.12	0.31	2.54	0.72	0.75	0.07	22.75
13	220	0.30	0.10	9.42	0.31	1.94	0.70	0.79	0.12	6.60
14	220	0.30	0.35	4.92	0.31	1.74	0.80	0.80	0.12	23.10
15	220	0.30	0.22	4.89	0.26	1.81	0.81	0.80	0.12	14.52
16	220	0.30	0.22	5.00	0.26	1.71	0.80	0.80	0.12	14.52
17	220	0.30	0.22	4.77	0.26	1.71	0.81	0.80	0.12	14.52
18	220	0.30	0.22	5.01	0.26	1.71	0.80	0.80	0.12	14.52
19	220	0.30	0.22	5.12	0.26	1.71	0.80	0.80	0.12	14.52

Source: Campos et al. [34]

Table 3 Coefficients of the RS models for T, R_a, and R_t

Coefficients	RS models		
	T (**x**)	R_a (**x**)	R_t (**x**)
β_0	4.963	0.260	1.733
β_1	− 0.861	− 0.007	0.048
β_2	− 3.055	0.028	0.278
β_3	− 1.440	0.000	− 0.052
β_{11}	1.115	0.005	− 0.010
β_{22}	1.456	0.000	0.092
β_{33}	0.756	0.018	0.021
β_{12}	1.060	− 0.010	− 0.054
β_{13}	0.918	− 0.008	− 0.029
β_{23}	1.435	0.005	− 0.021
R^2_{adj}	99.74%	98.66%	94.35%

Table 4 Industrial variables and their parameters

Stochastic variables	Unit	Symbol	Mean	St. Dev.
Setup time	min	t_p	60	6
Insert changing time	min	t_i	1	0.1
Batch size	pieces	Z	1000	100
Machine and labor costs	US$	$S_m + S_h$	50.00	5.00
Tool holder price	US$	K_{th}	125.00	12.50
Average tool holder life	edges	N_{th}	1000	100
Insert price	US$	K_i	31.25	3.13
Deterministic variables				
Secondary time	min	t_s	0.5	-
Tool approximation and retreat time	min	t_a	0.1	-
Number of cutting edges on the insert	units	N_i	4	-
Piece length	mm	l_f	50	-
Piece diameter	mm	D	49	-

study, so the correlations (ρ) among the seven stochastic variables were considered insignificant. Equation (11) was used to model the standard deviation of K_p, which leads to Eq. (15).

$$Var[K_p(\boldsymbol{\mu})] = \left[\frac{\partial K_p(\boldsymbol{\mu})}{\partial t_p}\right]^2 \sigma_{t_p}^2 + \left[\frac{\partial K_p(\boldsymbol{\mu})}{\partial t_i}\right]^2 \sigma_{t_i}^2 + \left[\frac{\partial K_p(\mu)}{\partial Z}\right]^2 \sigma_Z^2 + \left[\frac{\partial K_p(\boldsymbol{\mu})}{\partial (S_m + S_h)}\right]^2 \sigma_{(S_m+S_h)}^2 + \left[\frac{\partial K_p(\boldsymbol{\mu})}{\partial K_{\text{th}}}\right]^2 \sigma_{k_{\text{th}}}^2 + \left[\frac{\partial K_p(\mu)}{\partial N_{th}}\right]^2 \sigma_{N_{th}}^2 + \left[\frac{\partial K_p(\mu)}{\partial K_i}\right]^2 \sigma_{K_i}^2 \quad (15)$$

The reason why the other five variables were treated as deterministic is that their variances were considered insignificant compared with the seven stochastic variables for this study. Secondary time, in this particular case, consisted of 30 s only, so its variance was not relevant compared with the other variables. Tool approximation and retreat time are executed by the machine, so the variance among times was almost null. Because the same insert was used, the number of its cutting edges was in fact deterministic (always four cutting edges). The piece length and diameter did not present significant variances, as they were all supplied according to narrow specification limits.

Table 5 Partial derivatives of K_p on $\boldsymbol{\mu}$

Variables	Partial derivatives
t_p	$\frac{\partial K_p}{\partial t_p} = \frac{(S_m+S_h)}{60Z}$
t_t	$\frac{\partial K_p}{\partial t_i} = \frac{N_t^*(S_m+S_h)}{60Z}$
Z	$\frac{\partial K_p}{\partial Z} = \frac{(t_p - t_i)(S_m+S_h)}{60Z^2}$
$S_m + S_h$	$\frac{\partial K_p}{\partial (S_m+S_h)} = \frac{t_t}{60}$
K_{th}	$\frac{\partial K_p}{\partial K_{\text{th}}} = \frac{\mathrm{C_t}}{\mathrm{TN_{th}}}$
N_{th}	$\frac{\partial K_p}{\partial N_{\text{th}}} = \frac{C_t K_{\text{th}}}{TN_{\text{th}}2}$
K_i	$\frac{\partial K_p}{\partial K_i} = \frac{C_t}{TN_i}$

The partial derivatives of the stochastic variables are presented in Table 5, and the optimization problem is shown in Eq. (16),

$$\left\{ \max_x \int_{-\infty}^{\text{USL}} \phi \left\{ E\left[K_p(\mathbf{x}, \boldsymbol{\mu})\right], \quad \sqrt{\text{Var}\left[K_p(\mathbf{x}, \boldsymbol{\mu})\right]} \right\} \right. \begin{matrix} s.t.: \\ \mathbf{x}^T\mathbf{x} \le \rho^2 R_a(\mathbf{x}) \le \text{USL}_{R_a} \\ R_t(\mathbf{x}) \le \text{USS}_{R_t} \end{matrix} \quad (16)$$

where $\mathbf{x}$ is a vector composed of the decision variables, and $\boldsymbol{\mu}$ is a vector composed of the expected values of the seven stochastic industrial variables presented in Table 4. The goal was to determine the optimal levels for the machine parameters that present good results for K_p considering the variances of the industrial variables. More specifically, the optimization

Table 6 Results in optimal cutting conditions

Outputs					
C_t (min)	T_t (min)	T (min)	R_a (μm)	R_t (μm)	MRR (cm^3/min)
0.08	0.75	5.93	0.28	2.12	26.8

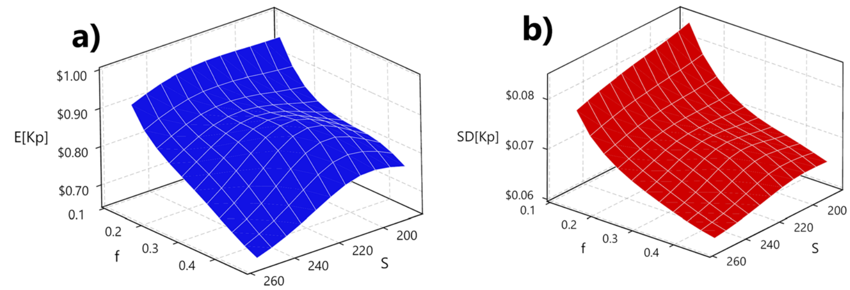

Fig. 2 Surface plot with V_c (S) and f for **a** $E[K_p]$ and **b** SD$[K_p]$

problem was to maximize the probability for the cost to be less than the USL of US$0.90, submitted to the experimental space and to the respective USLs of 0.8 μm for R_a and 4 μm for R_t, which correspond to the N6 ISO roughness grade number [37].

4 Results and discussion

4.1 Validation of $K_p^{'}$ variance

Before Eq. (16) was solved, the result of Eq. (15) was compared with a Monte Carlo simulation at the center points of the cutting conditions (Table 1). Equation (15) resulted in an expected value $E[K_p]$ = US$0.853 and a standard deviation SD$[K_p]$ = US$0.070. With the exact same conditions, Monte Carlo simulation with 10,000 replications resulted in $E[K_p]$ = US$0.852 and SD$[K_p]$ = US$0.070.

4.2 Problem solution

The generalized reduced gradient was used to solve the optimization problem presented in Eq. (16). The problem was built using Microsoft Excel software and its Solver add-in. The optimal levels of the decision variables were 240.9 m/min for cutting speed (V_c), 0.42 mm/rev for feed rate (f), and 0.26 mm for depth of cut (d). As a result, the 95% confidence interval (CI) for K_p was US$0.73 ± 0.12, and the maximum probability of K_p to be less than US$0.90 would be 99.71%. Other results are summarized in Table 6.

4.3 Effects of cutting conditions on K_p

Figures 2, 3, and 4 show the response surfaces for $E[K_p]$ and SD$[K_p]$ for each pair of decision variables. In each figure, the third decision variable was set in its optimal value. Figures 2 a, 3 a, and 4a show that the $E[K_p]$ varies significantly (from US$1.00 to US$0.66), depending on the levels of the decision variables. It is shown that a low expected value for K_p can be achieved by setting high values for cutting conditions. However, the values of SD$[K_p]$ did not vary as much (between US$0.06 and US$0.08), as shown in Figs. 2b, 3b, and 4b.

4.4 Effects of industrial variables on K_p

The individual results of the partial derivatives do not represent the impact of the industrial variables in a practical way, because they only represent their impact per unit. For instance, at the optimal cutting conditions, if t_i changes from 1 to 2 min (a 100% increase), then K_p rises approximately US$0.01. When it comes to $S_h + S_m$, if it becomes US$1.00 more expensive (only a 2% increase), then K_p also increases by US$0.01.

Therefore, to measure the real impact of the industrial variables, a full factorial design composed of 128 combinations was designed. Levels − 1 and + 1 were established considering a six sigma confidence interval, as shown in Table 7.

After calculating K_p for all combinations, machine and labor costs ($S_h + S_m$) were by far the most significant variable, followed by insert cost (K_i), setup time (t_p), and batch size (Z). If $S_h + S_m$ increases 30% (or three sigma), K_p increases US$0.19. Such an impact is

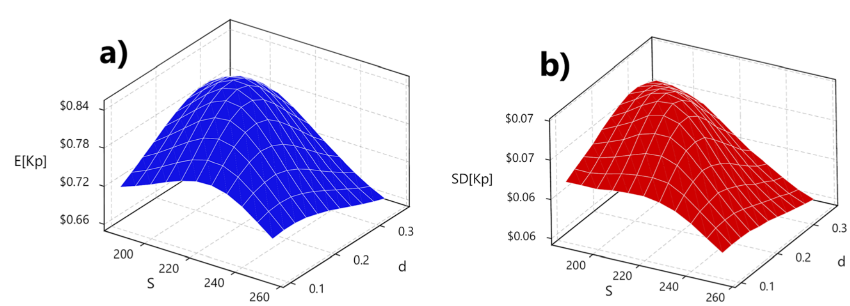

Fig. 3 Surface plot with V_c (S) and d for **a** $E[K_p]$ and **b** SD$[K_p]$

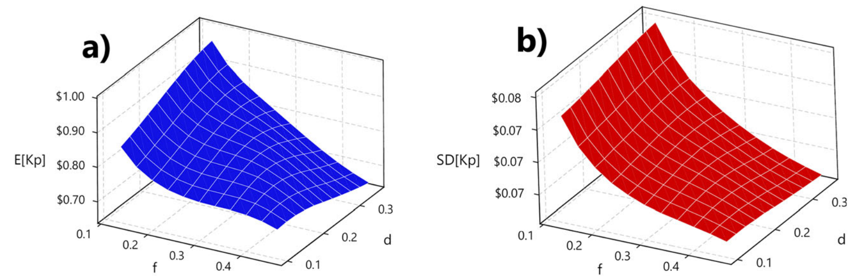

Fig. 4 Surface plot with f and d for **a** $E[K_p]$ and **b** $SD[K_p]$

10 times greater than that of the second-most-significant variable (K_i). The other variables alone did not present a significant impact. Yet, some two-variable interactions also had significant effects, such as t_p and Z, t_p and $S_h + S_m$, and Z and $S_h + S_m$.

It is also possible to estimate the potential impacts of improving some of the industrial variables. In this case study, if setup t_p was decreased from 60 to 9 min—a single-minute setup, as aimed at by single-minute exchange of dies (SMED) applications—and t_i was reduced to 0.5 min, K_p reduced 6.5% (from US$0.73 to US$0.68). Decision makers can use these estimations to analyze the feasibility of implementing SMED or other methodologies in their manufacturing processes, instead of focusing only on cutting parameters.

5.4. Minimal cost versus maximum tool life

The solutions of two optimization problems were compared: (a) Eq. (16) and (b) maximizing the tool life's response surface model [$T(\boldsymbol{x})$] alone by varying cutting conditions submitted to the same constraints, as shown in Eq. (17). Table 8 shows the solutions of both cases.

$$\begin{cases} \max_{x} T(\mathbf{x}) & \begin{array}{c} s.t.: \\ \mathbf{X}^T\mathbf{X} \leq \rho^2 \\ R_a(\mathbf{x}) \leq \mathrm{USL}_{R_a} \\ R_t(\mathbf{x}) \leq \mathrm{USL}_{R_t} \end{array} \end{cases} \quad (17)$$

The solution of Eq. (17) was a maximum tool life of 17.18 min. However, the 95% confidence interval for K_p was US$0.85 ± 0.15 for 95%, and the maximum probability for K_p to be less or equal to US$0.90 was 75.47%. More specifically, maximizing the tool life resulted in a 16.9% increase in $E[K_p]$ and a 18.9% increase in $SD[K_p]$.

To maximize the tool life, in Eq. (17), the levels of the decision variables (cutting speed, feed rate, and depth of cut) were set to much lower values compared with Eq. (16), as shown in Table 8. Tool life showed a 187.5% increase. However, with lower levels of cutting speed and feed rate, the cutting time (C_t) rose from 0.08 to 0.22 min, as in Eq. (3), which meant a 192.6% increase. In other words, the highest tool life would be useless, because the cutting time would increase as well. The number of tool changes (N_t) would be the same (12) within a batch size of 1000 pieces, and the average number of pieces cut by one cutting edge would be almost the same: 78 by solving Eq. (16) and 77 for Eq. (17).

Another important finding was that, after tool life was maximized, the total cycle time (T_t) increased from 0.748 to 0.894. This 19.5% increase results in an increase of K_p, as shown in Eq. (2).

Hence, maximizing the tool life by changing the cutting conditions does not necessarily reduce the process cost—it may actually increase it, as observed in this case study.

Table 7 Levels of the variables in the full factorial design

Variable	Level − 1	Level + 1	+ 3σ impact on K_p
t_p	30	90	U$ 0.02
t_i	0.7	1.3	U$ 0.00
Z	700	1300	− U$ 0.01
$S_m + S_h$	35.00	65.00	U$ 0.19
K_{th}	87.50	162.5	U$ 0.00
N_{th}	700	1300	− U$ 0.00
K_i	21.88	40.63	U$ 0.03

Table 8 Optimal cutting conditions and corresponding outputs

Problem	Cutting conditions					
	V_c (m/min)	f (mm/ver)	d (mm)	95% CI for K_p (U$)	T (min)	MRR (cm^3/min)
Eq. (16) − K_p ($\boldsymbol{x}$, $\boldsymbol{\mu}$)	240.9	0.42	0.26	0.73 ± 0.12	5.93	26.8
Eq. (17) − T ($\boldsymbol{x}$)	206.0	0.17	0.17	0.85 ± 0.15	17.18	5.7

5 Conclusions

The present study aimed to optimize the process cost of the AISI 52100 hardened-steel turning process. The cutting conditions were defined considering the impact of the main industrial variables on the cost. Stochastic programming was coupled with the RSM to represent the variables and formulate the optimization problem. The main results can be summarized as follows.

- The variance of a general continuous function was demonstrated and used to model the variance of the total process cost per piece (K_p). The demonstrated formula was validated using Monte Carlo simulation.
- Instead of creating unnecessary second-order polynomials and, thus, raising the variances of the models, direct formulas available in the literature were used to represent the cutting time (C_t), total cycle time (T_t), and K_p.
- The effects of cutting speed, feed rate, and depth of cut on K_p were measured, and response surfaces were plotted. These cutting conditions presented significant impacts on the expected value of K_p, but not on its standard deviation.
- The effects of the industrial variables on K_p were measured based on partial derivatives. At optimal cutting conditions, the most-significant variables on K_p were machine and labor costs, insert price, setup time, and batch size. If setup time is reduced from 60 to 9 min, for instance, K_p is expected to reduce 6.5%. Hence, instead of focusing only on cutting parameters, this study proves that industrial variables also have an impact on the process cost.
- The results of this particular case study also showed that maximizing tool life increased the process cost, because it required lower levels for the cutting conditions, which increased the C_t, T_t, and K_p.

It is planned to apply the stochastic programming method in this work to other processes in future research.

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