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# Stochastic optimization of AISI 52100 hard turning with six sigma capability constraint

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**ABSTRACT** Hard turning optimization problems are usually approached using Response Surface Methodology. By running designed experiments, researchers build analytical models to represent the outputs under interest. However, most studies focus on the expected values of the outputs, and only a few consider the variances of the models, even though there are several stochastic programming (SP) techniques available in the literature. Such variances may have a significant impact on the problem solution. This paper aims to optimize the AISI 52100 hardened steel turning process using SP. Decision variables are cutting speed, feed rate and depth of cut, outputs are cost per part and material removal rate, and average surface roughness six sigma capability is modeled as a stochastic constraint. The SP method is also compared to the conventional one, which did not include the variances into the problem. Results show that taking the variability of the models into account is necessary to obtain a satisfactory process capability and to analyze different scenarios. Finally, this study shows that competitive results can be achieved by simplifying the problem formulation.

**INDEX TERMS** Capability Engineering Optimization Methods, Response Surface Methodology, Stochastic Processes.

## I. INTRODUCTION

Hard turning has been of interest to both industry and academy due its particular characteristics and several advantages compared to conventional turning [1]. Such advantages include cost reduction, productivity increase, cycle time reduction, quality and material properties improvement, lower energy consumption and pollution reduction, as it eliminates coolant [2].

There is already a great number of recently published papers on hardened steel turning analysis and optimization. Some studies focus on the effects of process parameters (such as cutting speed, feed rate and depth of cut) on performance characteristics (i.e.: tool wear, surface roughness and material removal rate etc.) on AISI 52100 [3]–[6]. Others investigate the effects of inserts characteristics [7], [8]. Taguchi method is usually used for effects analysis. But for hard turning modeling and optimization, response surface methodology (RSM) is one of the most common techniques [9]. The optimization of industrial processes often includes two or more outputs [10], some of which may be strongly correlated. In this case, RSM has been combined with other methods such as multi criteria decision making [11],

principal component analysis (PCA) [12], or artificial neural networks (ANN) [13]–[15]. In particular, multivariate statistics are used when outputs are correlated [16]. This is an alternative to keep all the responses included in the problem.

Many of the aforementioned papers include a lot of correlated outputs in their optimization problem. However, multiobjective optimization problems may be simplified by selecting only a few key outputs instead of including all of the correlated outputs. Such approach may be suitable for industrial purposes, since what companies usually aim is to minimize costs, maximize productivity as long as quality requirements are achieved.

In order to use process capability, the variance of the outputs must be modeled along with their expected value. The coefficients of the response surface models are commonly estimated by ordinary least squares (OLS). These analytical models are stochastic, because their coefficients are normally distributed and represent a linear combination of the observations [11]. Nevertheless, most of the available studies on hard turning only use the expected values of the RSM model coefficients. The gap is to verify if the stochastic

nature of the models' coefficients is significant to the problem solution.

Within this context, this paper aims to optimize the AISI 52100 hardened steel turning process using stochastic programming. The main goals are: a) to formulate an optimization problem which is more related to industrial purposes; b) to quantify the impact of the variance of the output models on the results. In this study, the problem is to minimize cost per piece and maximize material removal rate, submitted to a six sigma average surface roughness capability. In addition, results of the present work are compared to the results of a previous study on hard turning optimization.

The paper is structured as follows: chapter II presents the method, which involved multiobjective optimization programming (MOP), stochastic programming coupled with MOP and process capability modeled as a stochastic constraint, chapter III details the experimental procedure and materials used, chapter IV presents the results, discussions are in chapter V and conclusions are in chapter VI.

## II. METHOD

The analysis and optimization of the AISI 52100 hardened steel turning started with the definition of a central composite design array and the experimental runs. After measuring the outputs under interest, analytical models were built using response surface methodology (RSM) to represent the expected values of the outputs under interest. In this work, two objective functions (cost and productivity) were defined and joined in a global function by Weighted Sums (WS) method using the formulation described in section A. Then, a stochastic constraint which defines a minimal process capability is established. Section B presents the general aspects of the stochastic programming technique and section C describes a specific formulation of a stochastic constraint of the process capability which includes the experimental data and the response surface models. Finally, the problem is solved using the general reduced gradient (GRG) method described in section D.

### A. MULTIOBJECTIVE OPTIMIZATION

Multiobjective Optimization Programming (MOP) can be defined as the formulation of problems whose objective is to optimize at least two results, which can be represented by analytical models  $f(x)$ , where  $x = [x_1, \dots, x_n]$  is a vector composed by the decision variables [17]. There are many strategies for MOP available in literature, among which there is the weighted sums (WS). For instance, Eq. (1) presents the general formulation for WS including two objective functions, where  $\bar{f}_1(x)$  and  $\bar{f}_2(x)$  are the standardized objective functions based on its utopian and Nadir values, where  $w$  is the weight of function 1 and  $g_j(x)$  refers to the problem constraints.

$$\begin{aligned} \text{Min } F(x) &= w\bar{f}_1(x) + (1-w)\bar{f}_2(x) \\ \text{s. t.:} & \end{aligned} \quad (1)$$

$$\begin{aligned} g_j(x) &\leq 0 \\ 0 &\leq w \leq 1 \end{aligned}$$

Eq. (2) presents the formula of a standardized model  $\bar{f}_i(x)$ . In this case, each model  $\bar{f}_i(x)$  is staggered based on its utopic value  $f_i^U(x)$  and its Nadir value  $f_i^N(x)$ . The utopic value is found by optimizing  $\bar{f}_i(x)$  alone, Nadir is the worst value of  $\bar{f}_i(x)$  among the other individual optimizations and  $f_i(x)$  is the current value of function  $i$ .

$$\bar{f}_i(x) = \frac{f_i(x) - f_i^U(x)}{f_i^N(x) - f_i^U(x)} \quad (2)$$

A payoff matrix  $P$  of two objective functions is presented in Eq. (3).

$$P = \begin{bmatrix} f_1^U(x) & f_1^N(x) \\ f_1^N(x) & f_2^U(x) \end{bmatrix} \quad (3)$$

### B. STOCHASTIC PROGRAMMING IN MOP

Stochastic programming (SP) consists of a strategy to build objective functions and constraints whose coefficients or decision variables are modeled as stochastic variables. The purpose of SP is to transform a stochastic problem into a deterministic one. The transformation depends intrinsically on the probability distributions used to represent the stochastic variables.

There are already different modeling strategies for stochastic programming (SP) proposed in the literature, including the E-model, V-model, P-model, minimax-methods [18] and stochastic DEA model [19]. Since in most MOP approaches the analytical models  $f_i(x)$  are usually obtained through Response Surface Methodology (RSM) [9], the SP method selected in the present study is specific for RSM.

After executing experiments according to a Central Composite Design array (CCD), results can be approximated to a second order polynomial model, as shown in Eq. (4).

$$\begin{aligned} Y \sim f(x) &= \beta_0 + \sum_{i=1}^k \beta_i x_i + \sum_{i=1}^k \beta_{ii} x_i^2 \\ &+ \sum_{i < j} \sum \beta_{ij} x_i x_j + \varepsilon = Z^T \beta + \varepsilon \end{aligned} \quad (4)$$

In Eq. (4),  $Y$  is the real response,  $Z$  is the vector composed by the terms (constant 1, linear, quadratic and interactions),  $\beta$  is a vector composed by the coefficients of each term and  $\varepsilon$  is the model error.

Coefficients in  $\beta$  are considered normally distributed  $\beta \sim N(\beta, \Sigma)$ , so it is reasonable to investigate if the variance and covariance matrix  $\Sigma$  causes a significant impact in the variance of  $f(x)$ .

Eq.s (5) and (6) present the formulas for  $\hat{\beta}$  and  $\Sigma$  respectively, where  $X$  is a matrix whose lines are composed by the values of  $Z^T$  in all the experimental runs. Proof of Eq.

(5) had already been presented in Rocha *et al.* [11], while Eq. (6) is found in Diaz-Garcia *et al.* [18].

$$\hat{\beta} = (X^T X)^{-1} (X^T Y) \quad (5)$$

$$\Sigma = \sigma^2 (X^T X)^{-1} \quad (6)$$

Finally, Eq. (7) presents the general formula to calculate the variance of the second order polynomial model as in Eq. (3) [16].

$$\text{Var}[f(x)] = Z^T \Sigma Z \quad (7)$$

### C. PROCESS CAPABILITY AS A STOCHASTIC CONSTRAINT

There are several ways to formulate an MOP problem including stochastic programming (SP). For instance, if a result has only an Upper Specification Limit (USL), which is the case for average surface roughness, its capability ( $C_{pk}|C_{pU}$ ) can be calculated by Eq. (8) [20].

$$C_{pk}|C_{pU} = \frac{USL - \mu}{3\sigma} \quad (8)$$

Based on Eq. (7), it is possible to formulate a constraint for a minimal process capability required in a process and add it to the MOP problem, as shows Eq. (9), where  $C_{p0}$  is the minimal capability required for the process.

$$\left[ \frac{USL^i - [Z^T \beta]}{3[\sqrt{Z^T \Sigma Z}]} \right] \leq C_{p0} \quad (9)$$

### D. GENERALIZED REDUCED GRADIENT ALGORITHM

The optimum values can be obtained by finding the stationary point of the fitted surface. The goal is to find the settings of  $x$ 's that optimize the objective function subject to the constraints. There are already several algorithms available in literature to solve such nonlinear optimization problems, including the generalized reduced gradient (GRG) [11], sequential quadratic programming (SQP), genetic algorithms (GA), simulated annealing, particle swarm and ant colony. The GRG is considered one of the most efficient and robust methods of constrained nonlinear optimization. The method is called "reduced gradient" because the algorithm works by substituting the constraints on the objective function and thus reducing the number of variables. Therefore, the number of gradients reduces [17].

The GRG method starts by classifying the original variables into basics ( $Z$ ) (dependents) and nonbasics ( $Y$ ) (independents). Then, one can write  $F(x) = F(Z, Y)$  and  $h(x) = h(Z, Y)$ . It is necessary that  $dh_j(x) = 0$  in order to meet the condition of optimality. So, if we define  $A = \nabla_Z h_j(x)$  and  $B = \nabla_Y h_j(x)$ , then we have  $dY = -B^{-1} A dZ$ . Hence, the GRG is defined as:

$$G_R = \frac{d}{dZ} F(x) = \nabla_Z F(x) - [B^{-1} A]^T \nabla_Y F(x)^T \quad (10)$$

The Searching direction is  $S_x = [-G_R dY]^T$  and we can use  $x^{k+1} = x^k + \alpha S^{k+1}$  to compute the interactions and to verify if  $x^{k+1}$  is adequate and  $h(x^{k+1}) = 0$  at each step. The last step of the method is the solution of  $F(x)$  as a function of  $\alpha$ , using a one-dimensional algorithm of search, as the Newton method for instance.

## III. APPLICATION: HARD TURNING PROCESS

In this chapter, the design of experiments is described (section A), followed by the experimental procedure (section B) and, finally, the multiobjective optimization problem proposed in this research.

### A. DESIGN OF EXPERIMENTS

Decision variables were cutting seed ( $S$ ), feed rate ( $f$ ) and depth of cut ( $d$ ). Experiments were carried out according to a central composite design (CCD): eight factorial points, six axial points and five center points, leading to a total of 19 runs. Eq. (11) was used to calculate the axial points distance to the center point ( $\rho$ ) for  $k$  number of factorial levels and  $n$  decision variables. The levels are shown in Table 1.

$$\rho = \sqrt[k]{k^n} = \sqrt[4]{2^3} = 1.682 \quad (11)$$

TABLE 1  
LEVELS AND VALUES OF THE DECISION VARIABLES

Coded	$S$ (m/min)	$f$ (mm/v)	$d$ (mm)
-1.682	186.4	0.13	0.10
-1	200.0	0.20	0.15
0	220.0	0.30	0.22
1	240.0	0.40	0.30
1.682	253.6	0.47	0.35

SOURCE: [21]

### B. EXPERIMENTAL PROCEDURE

A CNC Nardini Logic 175 lathe was used with a maximum rotation speed of 4000 rpm and a cutting power of 5,5 kW. The composition of the pieces of AISI 52100 used in the experiments was: 1.03% C, 0.23% Si, 0.35% Mn, 1.40% Cr, 0.04% Mo, 0.11% Ni, 0.001% S, 0.01%. After quenched and tempered, the pieces' hardness got between 49 and 52 HRC, up to a depth of 3 mm below surface. Wiper mixed ceramic inserts were used, coated with a thin layer of titanium nitride (TiN). The tool holder had a negative geometry with ISO code DCLNL 1616H12 and entering angle  $\chi_r = 95^\circ$ . More details of the materials and methods are found in [22]. Figure 1 shows the turning process of this study.

To measure tool life ( $T$ ), the wiper inserts were worn until their flank wear (VBC) indicator on the tool tip to reach 0.30 mm. This was the adopted criterion for the end of tool life and it was measured by an optical microscope. Cutting time ( $C_t$ ) was calculated by Eq. (12) for each experiment, according to the values of  $f$  and  $S$ . The pieces had a diameter  $d = 49$  mm and a length  $l_f = 50$  mm. Total cycle time ( $T_t$ ) was calculated by Eq. (13),

$$C_t = \frac{l_f \cdot \pi \cdot d}{1000 \cdot f \cdot S} \quad (12)$$

$$T_t = C_t + t_1 + t_2 \quad (13)$$

where  $t_1 = t_s + t_a + \frac{t_p - t_i}{Z}$  is the inproductive time and  $t_2 = \frac{C_t}{T} t_i$  is the tool changing time. After obtaining  $t_1$ ,  $t_2$ ,  $C_t$ , measuring tool life ( $T$ ) and using other data presented in Table 2, Eq. (14) can be used to calculate the total process cost per piece ( $K_p$ ).

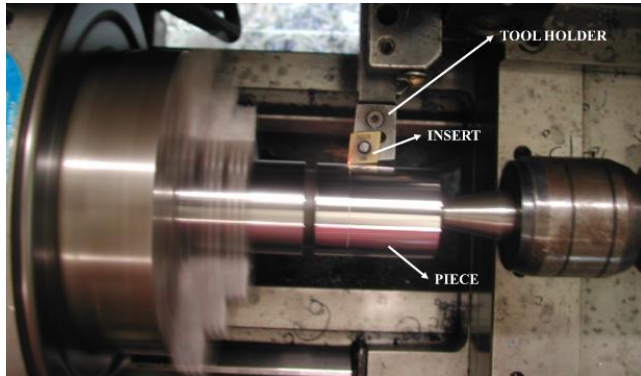


FIGURE 1. Turning process of AISI52100

TABLE 2  
DATA USED TO CALCULATE  $T_t$  AND  $K_p$

Variables	Symbol	Value
Secondary time	$t_s$	0.5 min
Tool approximation and retreat time	$t_a$	0.1 min
Set-up time	$t_p$	60 min
Insert changing time	$t_i$	1 min
Batch size	$Z$	1 unit
Machine and labor costs	$S_m + S_h$	US\$ 50.00
Tool holder price	$K_{th}$	US\$ 125.00
Insert price	$K_i$	US\$31.25
Average tool holder life	$N_{th}$	1 edge
Number of cutting edges on the insert	$N_i$	4 edges

SOURCE: [21]

$$K_p = \left( \frac{t_1}{60} - \frac{1}{Z} \right) (S_h + S_m) + \frac{C_t}{60} (S_h + S_m) + \frac{C_t}{T} \left[ \frac{K_{th}}{N_{th}} + \frac{K_i}{N_i} \right] + \frac{t_i}{60} (S_h + S_m) \quad (14)$$

Average surface roughness ( $R_a$ ) and maximum peak to valley roughness ( $R_t$ ) were measured for each wiper insert in its end of life. These responses were collected by a portable roughmeter with a cutoff length set to 0.8 mm. The measurements were taken at three different points of the workpiece. Each point was measured four times and the mean value among them was the final output.

Finally, material removal rate (MRR) was calculated by multiplying the decision variables  $S$ ,  $f$  and  $d$ . Table 3 shows the experimental data.

Instead of the six outputs, only three of them were initially considered in this study: total process cost per piece ( $K_p$ ), material removal rate (MRR) and average surface roughness ( $R_a$ ). The response surface model in Eq. (4) was used to model tool life ( $T$ ) and average surface roughness ( $R_a$ ). Vectors  $\beta$  were obtained using Eq. (5) and are shown in Table 4 along with the adjusted  $R^2$  for the models.  $K_p$  and MRR were directly calculated by formulas and thus did not require regression models.

### C. PROPOSED OPTIMIZATION PROBLEM (MOP 1)

The analytical models of  $K_p$  and MRR were standardized using Eq. (2) and agglutinated by Eq. (1). To do so, their utopic and Nadir values were required. Hence, each  $f_i(x)$  was individually optimized. The only restriction was related to the experimental space, as shows Eq. (15).

$$g(x) = \sqrt{S^2 + f^2 + d^2} \leq \sqrt[4]{k^n} = 1.682 \quad (15)$$

Table 5 presents results in a payoff matrix. The numbers in blue are utopic values and the red ones are Nadir values.

Process capability of  $R_a$  was modeled as a stochastic constraint using Eq. (9). In this case, the upper specification limit (USL) was established as 0.25  $\mu\text{m}$  and the minimal capability required ( $C_{p0}$ ) of 1.67. This value is usually defined for experiments because additional variability sources appear in the real systems, so  $C_p$  may be around 1 in practice [23]. Eq. (16) shows the proposed MOP problem.

$$\begin{aligned} \text{Min } F(x) &= wE[K_p] + (1 - w)E[MRR] \\ \text{s. t.:} \\ g_j(x) &\leq 0 \\ \frac{[0.25 - E[R_a]]}{[3\sqrt{\text{Var}[R_a]}}] &\geq 1.67 \\ 0 &\leq w \leq 1 \end{aligned} \quad (16)$$

### IV. RESULTS

In order to obtain the Pareto boundary, Eq. (16) was solved using the generalized reduced gradient (GRG) algorithm in Excel® Solver, with  $w$  varying from 0.05 to 0.95, in a 0.05 scale. Figure 2 presents the Pareto boundary for  $E[K_p]$  and  $E[MRR]$  using MOP 1. Along with the average curve, and considering a 95% confidence interval, two other ones were plotted: the best and worst scenarios. The best scenario uses the lower limit for  $K_p$  and upper limit for MRR, which means high productivity and low cost, respectively. The worst scenario is the opposite: upper limit for  $K_p$  and lower limit for MRR.

TABLE 3  
EXPERIMENTAL DATA

Run	Decision variables			Process outputs						
	$S$	$f$	$d$	$T$	$T_c$	$T_t$	$K_p$	$R_a$	$R_t$	$MRR$
1	200	0.20	0.15	17.21	0.19	0.86	0.76	0.25	1.41	6.0
2	240	0.20	0.15	11.37	0.16	0.83	0.76	0.27	1.72	7.2
3	200	0.40	0.15	5.96	0.10	0.77	0.72	0.31	2.12	12.0
4	240	0.40	0.15	4.48	0.08	0.76	0.72	0.30	2.15	14.4
5	200	0.20	0.30	9.42	0.19	0.87	0.84	0.25	1.45	12.0
6	240	0.20	0.30	7.37	0.16	0.84	0.82	0.25	1.58	14.4
7	200	0.40	0.30	4.03	0.10	0.78	0.79	0.34	2.01	24.0
8	240	0.40	0.30	6.10	0.08	0.75	0.68	0.29	1.99	28.8
9	186	0.30	0.22	9.51	0.14	0.81	0.74	0.29	1.69	12.3
10	254	0.30	0.22	6.86	0.10	0.77	0.71	0.26	1.81	16.8
11	220	0.13	0.22	14.18	0.27	0.95	0.89	0.21	1.54	6.3
12	220	0.47	0.22	4.12	0.07	0.75	0.72	0.31	2.54	22.8
13	220	0.30	0.10	9.42	0.12	0.79	0.70	0.31	1.94	6.6
14	220	0.30	0.35	4.92	0.12	0.80	0.80	0.31	1.74	23.1
15	220	0.30	0.22	4.89	0.12	0.80	0.81	0.26	1.81	14.5
16	220	0.30	0.22	5.00	0.12	0.80	0.80	0.26	1.71	14.5
17	220	0.30	0.22	4.77	0.12	0.80	0.81	0.26	1.71	14.5
18	220	0.30	0.22	5.01	0.12	0.80	0.80	0.26	1.71	14.5
19	220	0.30	0.22	5.12	0.12	0.80	0.80	0.26	1.71	14.5

SOURCE: CAMPOS ET AL. [21].

TABLE 4  
COEFFICIENTS OF THE RS MODELS

Coefficients	RS models	
	$T$	$R_a$
$\beta_0$	4.963	0.260
$\beta_1$	-0.861	-0.007
$\beta_2$	-3.055	0.028
$\beta_3$	-1.440	0.000
$\beta_{11}$	1.115	0.005
$\beta_{22}$	1.456	0.000
$\beta_{33}$	0.756	0.018
$\beta_{12}$	1.060	-0.010
$\beta_{13}$	0.918	-0.008
$\beta_{23}$	1.435	0.005
$R\text{-sq (adj)}$	99.74%	98.66%

TABLE 5  
PAYOFF MATRIX FOR  $K_p$  AND  $MRR$

	$K_p$	$MRR$
$K_p$	0.700	0.748
$MRR$	14.85	29.52

**A. CONVENTIONAL OPTIMIZATION (MOP 2)**

The proposed method was compared to modeling the results of interests by only their expected values  $E[f(x)]$ . Eq. (17) shows the proposed MOP problem and Fig. 3 presents the Pareto frontier for  $E[K_p]$  and  $s[K_p]$  for MOP 2.

$$\text{Min } F(x) = wE[K_p] + (1 - w) E[MRR]$$

s. t.:

$$g_j(x) \leq 0$$

$$E[R_a] \leq 0.25$$

$$0 \leq w \leq 1$$

(17)

**V. DISCUSSION**

At first, results seemed better in MOP 2. Fig. 3 shows material removal rates ( $MRR$ ) between 14.5 and 16.5 and costs per parts ( $K_p$ ) from 0.74 to 0.8. Meanwhile, Fig. 2 lower  $MRR$ s (from 12.5 to 14.5). As an example, let's consider the solution where  $w = 0.5$ , which means cost and productivity have the same importance. Results of MOP 1 and 2 are presented in Table 6.

Nevertheless, the variances of the models were not considered in MOP 2. For that reason, the capability results for  $R_a$  were null, because the expected value for  $R_a$  was always equal to its upper specification limit ( $USL$ ). It happened due the fact that the capability constraint was always active. But even if the decision makers set a minimal capability  $C_{pk_0}$  lower than the  $USL$ , they cannot be certain about the capability. The only way to guarantee that the capability is reasonable is by considering the models' variances like in MOP 1, where all solutions presented  $C_{pk} = 1.67$ .

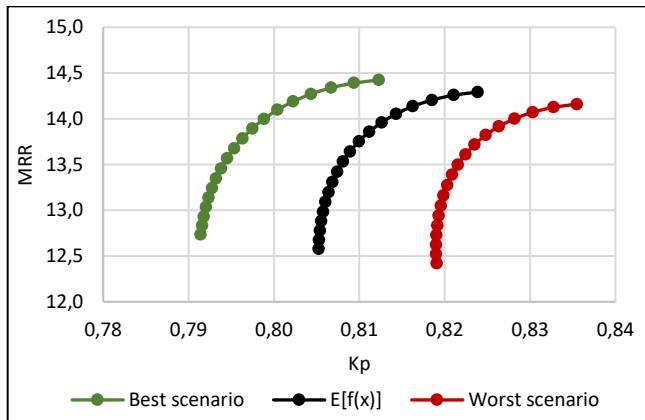


FIGURE 2. Pareto boundary for  $K_p$  and  $MRR$  using MOP 1

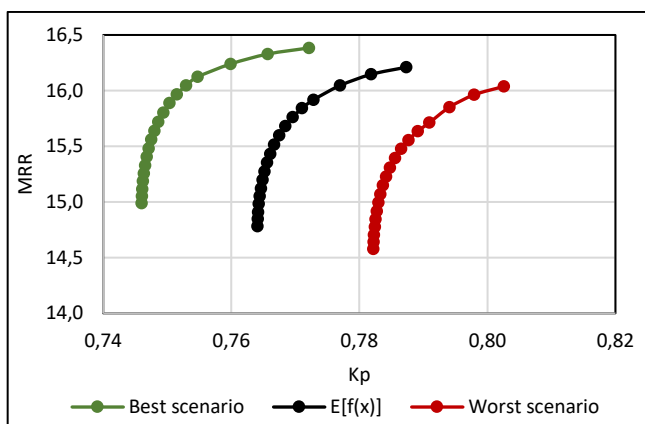


FIGURE 3. Pareto Boundary for  $K_p$  and  $MRR$  using MOP 2

TABLE 6  
RESULTS OF MOPS 1 AND 2 FOR  $w = 0.5$

Output	MOP 1	MOP 2
$E [K_p]$	0,81	0,766
$E [MRR]$	13,5	15,432
$E [R_a]$	0,243	0,25
$C_{Pk} [R_a]$	1,67	0

The problem was also modeled using normal boundary intersection (NBI) method. However, the Pareto boundary was twisted and did not present equally spaced solutions. It happened because the stochastic constraint of  $R_a$  capability was always active, so the solutions found by the algorithm got twisted from each other. On the other hand, when the stochastic constraint was changed until it was no longer active, the Pareto boundary were equally spaced and no longer twisted. Although, when the process capability is no longer a concern, then using an MOP with stochastic programming would not be relevant in the first place. Finally, we compare the results of the present study to the results achieved in a previous research [17], as shown in Table 7. Cutting conditions were significantly different: while in the previous study the cutting speed, feed rate and depth of cut were high, in the current study these variables were kept in

lower values. Consequently,  $MRR$  decreased significantly, since it is the multiplication of the three cutting conditions. Tool life ( $T$ ) was higher in the previous study, however it did not cause any significant impact on  $K_p$ . The reason is that for  $T$  to be increased, lower levels of cutting speed, feed rate and depth of cut are usually required. By doing so, cutting time ( $C_t$ ) and total cycle time ( $T_t$ ) increase, along with labor and machine costs per piece. A proof of it is that a maximum tool life does not necessarily mean that tool wear is maximized. In this case, the number of tool changes was the same in the two cutting conditions.

TABLE 7  
RESULTS COMPARISON TO PREVIOUS WORK

Variable	Units	Previous study [17]	Current
$S$	m/min	250	204
$f$	mm/rev	0.25	0.22
$d$	mm	0.26	0.19
$T$	min	7.19	12.21
$C_t$	min	0.13	0.17
$T_t$	min	0.81	0.84
$K_p$	US\$	0.76	0.76
$R_a$	$\mu\text{m}$	0.25	0.24
$R_t$	$\mu\text{m}$	1.67	1.51
$MRR$	cm <sup>3</sup> /min	16.25	8.43

Similar results were achieved for  $R_a$  and  $R_t$ . The difference, however, is that process capability was estimated only in the current study. In addition, the previous research modeled  $R_a$  and  $R_t$  as objective functions, but in industry, these outputs are constraints and should be modeled as such. Otherwise, they will compete with other conflicting objective functions, increasing the difficulty of problem solution unnecessarily.

## VI. CONCLUSIONS

The present study optimized the AISI 52100 hardened steel turning process using stochastic programming. The input variables were cutting speed, feed rate and depth of cut, the objective functions were cost per part and material removal rate and the capability of the average surface roughness was modeled as a stochastic constraint.

The proposed method (MOP 1) was applied and then compared to the traditional approach (MOP 2), where the variance is not modeled. Results showed that, differently from MOP 2, MOP 1 could guarantee a minimal process capability. Best and worst scenarios were also plotted in the Pareto boundary, for a 95% confidence interval. In this specific case, normal boundary intersection (NBI) presented some difficulties in providing a satisfactory Pareto boundary. Finally, results between this study and the addressed one were compared. Competitive results were achieved in the current paper by modeling the problem in a simpler manner and in a closer perspective to industrial reality.

Future research may consider the stochastic nature of many other input variables that are related to the cost per part. Another possibility is to optimize not only the expected values of the models, but also minimize their variances, and use multivariate statistics such as PCA in case of highly correlated responses.

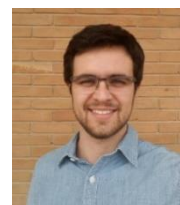
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## REFERENCES

- [1] Y. Huang, Y. K. Chou, and S. Y. Liang, "CBN tool wear in hard turning: A survey on research progresses," *Int. J. Adv. Manuf. Technol.*, vol. 35, no. 5–6, pp. 443–453, 2007.
- [2] V. N. Gaitonde, S. R. Karnik, L. Figueira, and J. Paulo Davim, "Machinability investigations in hard turning of AISI D2 cold work tool steel with conventional and wiper ceramic inserts," *Int. J. Refract. Met. Hard Mater.*, vol. 27, no. 4, pp. 754–763, 2009.
- [3] K. Bouacha, M. A. Yaltese, S. Khamel, and S. Belhadi, "Analysis and optimization of hard turning operation using cubic boron nitride tool," *Int. J. Refract. Met. Hard Mater.*, vol. 45, pp. 160–178, 2014.
- [4] I. Meddour, M. A. Yaltese, R. Khattabi, M. Elbah, and L. Boulanour, "Investigation and modeling of cutting forces and surface roughness when hard turning of AISI 52100 steel with mixed ceramic tool: cutting conditions optimization," *Int. J. Adv. Manuf. Technol.*, vol. 77, no. 5–8, pp. 1387–1399, 2015.
- [5] W. Bin Rashid, S. Goel, J. P. Davim, and S. N. Joshi, "Parametric design optimization of hard turning of AISI 4340 steel (69 HRC)," *Int. J. Adv. Manuf. Technol.*, vol. 82, no. 1–4, pp. 451–462, 2016.
- [6] M. Mia *et al.*, "Taguchi S/N based optimization of machining parameters for surface roughness, tool wear and material removal rate in hard turning under MQL cutting condition," *Measurement*, vol. 122, pp. 380–391, 2018.
- [7] S. A. Khan, M. Umar, M. Q. Saleem, N. A. Mufti, and S. F. Raza, "Experimental investigations on wiper inserts' edge preparation, workpiece hardness and operating parameters in hard turning of AISI D2 steel," *J. Manuf. Process.*, vol. 34, no. December 2017, pp. 187–196, 2018.
- [8] R. Kumar, A. K. Sahoo, P. C. Mishra, and R. K. Das, "Comparative study on machinability improvement in hard turning using coated and uncoated carbide inserts: part II modeling, multi-response optimization, tool life, and economic aspects," *Adv. Manuf.*, vol. 6, no. 2, pp. 155–175, 2018.
- [9] D. C. Montgomery, *Design and analysis of experiments*, 8th ed. Hoboken, NJ: John Wiley & Sons, 2017.
- [10] M. Junaid Mir and M. F. Wani, "Modelling and analysis of tool wear and surface roughness in hard turning of AISI D2 steel using response surface methodology," *Int. J. Ind. Eng. Comput.*, vol. 9, no. 1, pp. 63–74, 2018.
- [11] L. C. S. Rocha, A. P. de Paiva, P. Rotela Junior, P. P. Balestrassi, and P. H. da Silva Campos, "Robust multiple criteria decision making applied to optimization of AISI H13 hardened steel turning with PCBN wiper tool," *Int. J. Adv. Manuf. Technol.*, vol. 89, no. 5–8, pp. 2251–2268, 2017.
- [12] R. S. Peruchi, P. Rotela Junior, T. G. Brito, J. J. J. Largo, and P. P. Balestrassi, "Multivariate process capability analysis applied to AISI 52100 hardened steel turning," *Int. J. Adv. Manuf. Technol.*, vol. 95, no. 9–12, pp. 3513–3522, 2018.
- [13] A. Labidi, H. Tebassi, S. Belhadi, R. Khattabi, and M. A. Yaltese, "Cutting Conditions Modeling and Optimization in Hard Turning Using RSM, ANN and Desirability Function," *J. Fail. Anal. Prev.*, vol. 18, no. 4, pp. 1017–1033, 2018.
- [14] I. Meddour, M. A. Yaltese, H. Bensouilah, A. Khellaf, and M. Elbah, "Prediction of surface roughness and cutting forces using RSM, ANN, and NSGA-II in finish turning of AISI 4140 hardened steel with mixed ceramic tool," *Int. J. Adv. Manuf. Technol.*, vol. 97, no. 5–8, pp. 1931–1949, 2018.

- [15] P. Y. Chou, J. T. Tsai, and J. H. Chou, "Modeling and Optimizing Tensile Strength and Yield Point on a Steel Bar Using an Artificial Neural Network with Taguchi Particle Swarm Optimizer," *IEEE Access*, vol. 4, pp. 585–593, 2016.
- [16] R. A. Johnson and D. W. Wichern, *Applied Multivariate Statistical Analysis*, 6th ed. Upper Saddle River, 2007.
- [17] S. S. Rao, *Engineering Optimization: theory and practice*, 4th ed. New Jersey: Wiley, 2009.
- [18] J. A. Díaz-garcía, R. Ramos-quiroya, and E. Cabrera-Vicencio, "Stochastic programming methods in the response surface methodology," *Comput. Stat. Data Anal.*, vol. 49, pp. 837–848, 2005.
- [19] M. Wen, T. Zu, M. Guo, R. Kang, and Y. Yang, "Optimization of Spare Parts Varieties Based on Stochastic DEA Model," *IEEE Access*, vol. 6, pp. 22174–22183, 2018.
- [20] D. C. Montgomery and G. C. Runger, *Applied statistics and probability for engineers*, 6th ed. New York: John Wiley & Sons, 2013.
- [21] P. H. S. Campos *et al.*, "Multivariate mean square error for the multiobjective optimization of AISI 52100 hardened steel turning with wiper ceramic inserts tool: a comparative study," *J. Brazilian Soc. Mech. Sci. Eng.*, vol. 39, no. 10, pp. 4021–4036, 2017.
- [22] P. H. da S. Campos, "Otimização Robusta Multivariada do Processo de Torneamento do Aço Endurecido ABNT 52100 com Ferramenta Cerâmica Alisador," Universidade Federal de Itajubá, 2011.
- [23] J. M. Juran and F. M. Gryna, *Quality Planning and Analysis: From Product Development Through Use*, 3rd ed. McGraw-Hill, 2001.



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