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A Multivariate Descriptor for Change Point Detection in Nonlinear Time Series

Abstract— **In this paper a novel method is applied to detect dynamic changes in nonlinear time series. A multivariate control chart monitoring special causes of variation on the normalized descriptors of activity, mobility and complexity was applied to a change point detection problem on six synthetic nonlinear time series. The method was quite competent on the recognition of dynamic changes of shift and volatility for the time series. A case study on six series of short-term electricity load consumption was used to corroborate the results.**

Index Terms— **Nonlinear Time Series, Hjorth Descriptors, Hotelling Control Chart, Change Point**

I. Introduction

THE present work proposes a novel method to detect dynamic changes in nonlinear time series using a multivariate normalized descriptor. This detection is of fundamental importance for several applications when nonlinear time series are recorded. Nonlinear time series are found in many applications such as electricity load, EEG signals, econometric time series, water consumption, etc. The detection of dynamic changes is related to the management of special causes of variation that often are related to process problems.

A multivariate control chart monitoring special causes of variation on the normalized descriptors of activity, mobility and complexity was applied to detect changes on six synthetic and biased nonlinear time series. The method was able to recognize the dynamic changes in less than half of the seasonality considered for the time series, which is a reasonable time for many real applications. A case study on six series of electricity load consumption was used to corroborate the results.

This paper describes in section II the recent literature review on the detection of dynamic changes for problems of nonlinear time series. The multivariate method is also presented and a follow-along example shows the calculation for the proposed methodology. Section III presents numerical and graphical results for the overall procedure. Section IV presents a case study for short-term electricity load problem using the novel framework. Section V states the main conclusions and further research.

II. BACKGROUND AND LITERATURE REVIEW

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A. Detecting Changes in Nonlinear Time Series

Temporal variations in process parameters are often both complex and nonstationary. While nonlinear reconstruction and other approaches can be used to attempt analysis of such data, it is sometimes only necessary to detect substantial changes in the underlying process. In monitoring a sequence of correlated observations over time, a key issue is to detect a change in the underlying process structure as quick as possible, while controlling the rate of false alarms at a fixed level. Quick detection of change(s) enables users to make an immediate decision to control a system dynamically because it allows them to react quickly and make necessary adjustments in a timely manner. For example, there is evidence that epileptic seizures are preceded by changes in EEG signals and that seizure prediction may be possible. In a sequential manufacturing process, for example, a product unit proceeds through different manufacturing stages. At these stages, sensors monitor the features of the unit to detect abrupt changes in process variables as well as to forecast their future behavior. The information thus received could be further utilized in the preventive maintenance and quality control. These kinds of problems arise in many fields.

The problem of detecting break points for a broad class of non-stationary time series models have been investigated in the literature. This is a classical problem in signal processing, which can be used for example for event detection. References [2] to [10] present several methods involving Sequential Algorithms, Wavelet decomposition, Markov Chain Monte Carlo method, Bayesian approach, Sequential Probability Ratio test, CUSUM control chart, Neural Network, etc…

Much prior work on this problem has focused on methods for detecting changes in the mean and/or variance structure of a time series, most often using a time domain approach. For example, [11] and [12] discuss time domain methods for detecting multiple changes in the variance of a time series, with application to a time series of stock returns. Reference [13] provides extensive references on change point detection methods for correlated data. Online methods for detecting changes in the second order properties of the process received less attention. For retrospective change point detection [14] presents a test based on the cumulative integrated periodogram. Reference [15] presents a parametric approach to online change point detection for time series based on fitting autoregressive (AR) models to blocks of series and comparing them using likelihood-based methods. Although AR models can capture a wide-range of process behavior, the results of the methodology are likely to be poor if the structure differs significantly from that of an AR process. Nonparametrics methods, such as those based on spectral properties of the process, have not been examined in the online setting.

However, the current change point detection methods tend to be quite sophisticated in nature. Additionally, a priori knowledge about the possibilities of the changes and their distributions is often required. This may make the implementation of these methods difficult as an automatic, on-line change detection application. Some drawbacks are also the requirement of several parameters estimation, the need of several descriptors monitoring and the great number of variables tuning. The concerns are increased when multiple time series are used at the same time.

The main contribution of this work is the online change point detection methods that are based on the idea that changes in the second order structure of a stochastic process are reflected by changes in the Fourier or Wavelet-based spectra, which are the decompositions of variance across frequency and scale, respectively. By partitioning the series into short blocks, so that the stationary can be safely assumed, this work develops a spectral-based method for comparing the process behavior across blocks, to determine whether a change occurred. As mentioned above, an advantage of the spectral-based methods is that they do not assume a particular process structure, such as AR, and thus can be effective for series with non-AR or/and non-MA (Moving Average) spectra.

B. The Multivariate Approach

Hjorth Descriptors

In 1970, Hjorth formulated three parameters capable of characterizing any signal and its derivatives in the frequency and time domains. The Hjorth parameters are called normalized slope descriptors because they can be defined as first and second derivatives. They are named: Activity (a measure of the mean power), Mobility (an estimate of the mean frequency) and Complexity (an estimate of the frequency spread and bandwidth) [16]

By definition, activity (m_o) mobility (m_2) and complexity (m_4) are represented by the following formulas:

$$m_0 = \int_{-\infty}^{+\infty} S(w)dw = \frac{1}{T}\int_{t-T}^{t} x^2(t)dt \tag{1}$$

$$m_2 = \int_{-\infty}^{+\infty} w^2 S(w)dw = \frac{1}{T}\int_{t-T}^{t} \left(\frac{df}{dt}\right)^2 dt \tag{2}$$

$$m_4 = \int_{-\infty}^{+\infty} w^4 S(w)dw = \frac{1}{T}\int_{t-T}^{t} \left(\frac{d^2x}{dt^2}\right)^2 dt \tag{3}$$

where m_n is the spectral moment of order n, $S(w)$ in the power density spectrum in radians and $x(t)$ is the nonlinear time series as a function of continuous time.

Their computation in discrete time involves the variance ($\sigma_0^{\ 2}$) of $x[n]$ (the segment of the nonlinear time series to be analyzed) as well as the variance of the first and second derivatives of $x[n]$ ($\sigma_1^{\ 2}$ and $\sigma_2^{\ 2}$ respectively). These measures are described by the following formulas:

$$A = Activity = m_0 = \sigma_0^{\ 2} \tag{4}$$

$$M = Mobility = m_2 = \frac{\sigma_1}{\sigma_0} \tag{5}$$

$$C = Complexity = m_4 = \sqrt{\left(\frac{\sigma_2}{\sigma_1}\right)^2 - \left(\frac{\sigma_1}{\sigma_0}\right)^2} \tag{6}$$

Given that Hjorth parameters are based on variances, the computational cost is affordable to other methods. In this study, Hjorth parameters were used to discriminate distinct stages, were the dynamic system changes overtime.

The harmonic Hjorth parameter, which are frequency-domain versions of the Hjorth parameters are: the center frequency (f_c), the bandwidth (f_σ) and the value at the central frequency (Pf_c). Their calculation requires power spectrum estimation $P(f)$ of the epoch and they are defined as follows:

$$f_c = \int_{f_L}^{f_H} fP(f)df \Big/ \int_{f_L}^{f_H} P(f)df \tag{7}$$

$$f_\sigma = \sqrt{\int_{f_L}^{f_H} (f - f_c)^2 P(f)df \Big/ \int_{f_L}^{f_H} P(f)df} \tag{8}$$

$$Pf_c = P(f_c) \tag{9}$$

Since the computation of $P(f)$ involves discrete values, the above formulas were approximated using summations as follows:

$$f_c = \sum_{f=f_L}^{f_H} f\hat{P}(f) \Big/ \sum_{f=f_L}^{f_H} \hat{P}(f) \tag{10}$$

$$f_\sigma = \sqrt{\sum_{f=f_L}^{f_H} (f - f_c)^2 \hat{P}(f) \Big/ \sum_{f=f_L}^{f_H} \hat{P}(f)} \tag{11}$$

$$Pf_c = \hat{P}(f_c') . \tag{12}$$

In the above formulas, the index spans from the lowest to the highest time series frequency.

Multivariate Control Charts

Multivariate control charts are designed to incorporate the correlation structure of several variables into a chart when determining the state of statistical control of a process. Hotelling T^2, multivariate CUSUM and multivariate EWMA are examples of such a control charts designed to detect changes in the mean vector of several related quality characteristics under a constant covariance matrix, Σ. The interested reader are referred to [17], a valuable resource for multivariate control chart.

The statistic calculated for use in the Hotelling T^2 chart, adopted in this study is:

$$T^2 = n(\bar{x}_i - \bar{\bar{x}})' \bar{S}^{-1} (\bar{x}_i - \bar{\bar{x}}) \quad (13)$$

For $i=m+1,\ldots$, where $\bar{\bar{x}} = \frac{1}{m}\sum_{i=1}^{m} \bar{x}_i$ is the sample mean and $\bar{S} = \frac{1}{m}\sum_{i=1}^{m} S_i$ is the sample covariance matrix from an in-control, historical sample of m subgroups of size n for monitoring p variables. The distribution of the T^2 statistic is closely related to the F distribution using the following:

$$T^2 \approx \frac{p(m+1)(n+1)}{n(mn-m-p+1)} F_{\lambda,p,mn-m-p+1} \quad (14)$$

where λ is the noncentrality parameter calculated as:

$$\lambda = (\mu_1 - \mu_0)' \Sigma^{-1} (\mu_1 - \mu_0) \quad (15)$$

where μ_0 is the in-control mean vector and μ_1 is the current mean vector of the process variables. This parameter is sometimes referred to as the signal-to-noise ratio. Note that when the process is in control $\mu_0 = \mu_1$ and $\lambda = 0$. Based on this relationship, the control limit for the T^2 control chart is based on an upper-tail percentile from the central F distribution to obtain the desired in-control Average Run Length (ARL). When the sample size is one and the process is in-control, the T^2 distribution is related to the F distribution using

$$T^2 \approx \frac{k(n+1)(n-1)}{n(n-k)} F_{k,n-k} \quad (16)$$

where k is the number of variables.

C. A Follow-Along Example

The following simplified example shows how the proposed methodology is applied. The example considers a nonlinear Smoothing Autoregressive (SAR) time series described by the following model:

$$y_t = sign(y_{t-12}) + \varepsilon_t \quad (17)$$

where $sign(x)$=1, 0, -1 if $x>0$, $x=0$ and $x<0$, respectively. The error term ε_t follows a normal distribution $N(0,1)$.

The nonlinear time series was divided into two segments. The first segment with 24 points represents the pure SAR model. The second segment considers a change point where the mean was increased by two standard deviation. Fig. 1 shows these segments.

[INSERT FIG. 1 HERE]

The Hjorth's descriptors, A, M and C were computed using (4), (5) and (6). The first and second derivatives were obtained using backward differentiation:

$$f'(x_i) = \left(\frac{f(x_i) - f(x_{i-1})}{\Delta x} \right) - error \quad (18)$$

$$error = \frac{(\Delta x)}{2!} f''(x_i) + \frac{(\Delta x)^2}{3!} f'''(x_i) + \ldots \quad (19)$$

The idea of moving window was used to calculate all the point statistics. This means that all the statistics for t=24, were obtained considering the points 1 to 24. The statistics for t=25 have considered points 2 to 25 and so on. The correlation between the Hjorth descriptors were considered significant: r_{AM}=-0.901, r_{AC}=0.868 and r_{MC}=-0.923. Fig. 2 shows the Hjorth's descriptors.

[INSERT FIG. 2 HERE]

The Hotelling T^2 control chart points were calculated using (13) and (16). An out-of-control point was found on the control chart (point 25) based on the most popular criteria of falling over 3 standard deviation. The decomposed value associates the causes of Activity and Mobility to the outlier. The Hotelling T^2 control chart is shown on Fig. 3.

[INSERT FIG. 3 HERE]

The interested reader can reproduce the proposed methodology by computing the results on Table I.

[INSERT TABLE I HERE]

III.SIMULATION RESULTS

A. Nonlinear Time Series: The dataset

Linear time series methods have been widely used to represent process for the past two decades. Recently, however, there has been increasing interest in extending the classical framework of Box and Jenkins [18] to incorporate nonstandard properties, such as nonlinearity, non-Gaussianity, and heterogeneity. In this way, a great number of nonlinear models have been developed, such as the bilinear model [19], the threshold autoregressive (TAR) model [20], the state-dependent model [21], the Markov switching model [22], the functional-coefficient autoregressive model [23], among many others. Although the properties of these models tend to overlap somewhat, each is able to capture a wide variety of nonlinear behavior. This kind of time series is even more complex due to some features like high frequency, daily and weekly seasonality, calendar effect on weekend and holidays, high volatility and presence of outliers.

Consider a univariate time series xt which for simplicity is observed at equally spaced time points. We denote the observations by {xt|t=1,...,T}, where T is the sample size. A purely stochastic time series xt is said to be linear if it can be written as

$$x_t = \mu + \sum_{i=0}^{\infty} \psi_i a_{t-i} \quad (20)$$

where μ is a constant, ψ_i are real numbers with $\psi_0 = 1$, and {at} is a sequence of independent and identically distributed (iid) random variables with a well-defined distribution function. We assume that the distribution of at is continuous and E(at)=0. In many cases, we further assume that Var(at)= σ_a^2 or, even stronger, that at is Gaussian. If $\sigma_a^2 \sum_{i=1}^{\infty} \psi_i^2 < \infty$ then xt is weakly stationary (i.e., the first two moments of xt are time-invariant). The ARMA process is linear because it has an MA representation in the mentioned equation. Any stochastic process that does not satisfy the condition of this equation is said to be nonlinear (see [25] for more details). For previous and more general surveys on nonlinear time series models, the interested reader is referred to [26] and [27].

Nonlinear models define different states of the world or regimes, allow the possibility that the dynamic behavior of variables depends on the regime that occurs at any given point in time [21]. By state-dependent dynamic behavior of a time series it is meant that certain properties of the time series, such as its mean, variance and/or autocorrelation, are different in different regimes. In particular, autocorrelations tend to be larger during periods of low volatility and smaller during periods of high volatility. The periods of low and high volatility can be interpreted as different regimes. The state-dependent, or regime-switching considers that the regime is stochastic and not deterministic, witch is relevant for many time series (mainly financial). There are two main classes of regime switching models: (i) Regimes determined by observable variables that include the Bilinear model, the TAR (Threshold Autoregressive) model and the SETAR (Self-Exciting Threshold Autoregressive) model and (ii) Regimes determined by unobservable variables that include the MSW(Markov-Switching) model. We restrict our attention to models that assume that in each of the regimes the dynamical behavior of the time series is modeled with an AR model. In other words, the time series is modeled with an AR model, where the autoregressive parameters are allowed to depend on the regime or state. Generalizations of the MA model to a regime-switching context have also been considered [27], but we abstain from discussing these models here.

Table I shows a collection of nonlinear time series implemented and simulated for the present study. In each case, ε_t:N(0, 1) is assumed to be *iid*. These eight time series models are chosen to represent a variety of problems which have different characteristics. For example, some of the series have pure autoregressive (AR) or pure moving average (MA) correlation structures while others have mixed AR and MA components. Similar models (with different lags) were explored in [28].

[INSERT TABLE II HERE]

B. Results

The main results of the simulation study are presented in Table III where the Average Run Length (ARL) was computed for the six nonlinear time series. The ARL shows the number of samples needed to detect a process shift (a change point) in the time series. Two kinds of shift were here considered: the mean shift and the standard deviation shift. The control limits for the Hotelling T^2 control chart were defined by the in-control sample (the first 24 points). The results were obtained considering 100 simulations for each time series. A user friendly Statistica [29] macro was developed to run the simulation. For each run, a moving window of 24 points was processed. In this way, the first control chart used the in-control points at t_1 to t_{24}. The second control chart added one point (t_{25}) under a new regime and removed the point at t_1. In this way the second control chart process the points at t_2 to t_{25}, the third chart the points at t_3 to t_{26} and so on. This process is repeated until an out-of-control point is obtained. When an outlier is found after introducing the new regime at t_{ARL}, the ARL value is obtained subtracting t_{ARL}-24. It is desirable high values of ARL when the time series is in control and low values of ARL when the time series change.

[INSERT TABLE III HERE]

The obtained ARLs in Table III for all time series is comparable to traditional results related to an Xbar control chart. ARL for large shifts (both in mean and/or in variance) are very low. This means that the change point detection is obtained with just a few points after the introduction of a new regime. ARLs without shift are very large meaning that false alarms are rarely obtained. A paired t-test comparing the ARLs for the lags 12 and 24 resulted in a p_value<0.05. This means that change point in nonlinear time series with small seasonality is detected faster than with large seasonality.

For the Nonlinear Moving Average (NMA) time series, the ARLs correspondents of false alarms (without shift) were statistically different when compared to the others time series. When compared to traditional control charts the results were satisfactory. This time series, with only nonlinear moving average realizations, represented the worst case scenario for the simulation results because false alarms were obtained with fewer points. This is related to the high volatility of this time series.

Fig. 4 shows the multi-vari chart for the results on Table III, considering averaged values. It can be observed that the mean shift and sigma shift were significant factors for ARLs. Lag was not statistically significant.

[INSERT FIG. 4 HERE]

IV. CASE STUDY: SHORT-TERM ELECTRICITY LOAD

Companies that trade in deregulated electricity markets in the US, Brazil, England, and most other countries use time series forecasting to predict demand of electricity as part of the information necessary to set buying and selling contracts.

If they are production or trade companies, their interests lie particularly in optimizing the energy load and prices sold to their costumers.

This case study deals with the modeling and forecasting of an important variable that affects achieving a good portfolio of electricity contracts: the electricity load (or consumption). Knowledge of its future value and its variation is essential to calculate the portfolio risk and return. Although it is possible to contract a certain amount of energy with no allowed variation, for a consumer it represents high risk. In the industrial process there are a lot of unpredictable factors that can cause an increase or decrease in energy consumption. If an electricity producer can accept and manage this risk, the producer will have more opportunities in the market. Thus it is important to manage these risks.

As considered by [30] and [31], forecasting electricity prices and loads is an arduous task due to a great number of factors such as electricity demand and supply, number of generations, transmission and distribution constraints, etc. For deregulated electricity markets, a great volume of electricity is bought and sold in the spot market as well as in the bilateral market among agents at different geographical regions. A simple unpredictable climate change can add volatility in demand and therefore volatility in prices. Reference [32] also state that electricity load forecasting is complex to conduct due to nonlinearity of its influenced factors. They state that the most important factor in regional or national power system strategy management has been accurate load forecasting of future demand.

This case study intends to use the present framework to the change point detection of electricity load of six industrial consumers in Brazil using historical hourly time series. The change point here represents the time when the consumers goes to the spot market and can test the consumer's fidelity.

Fig. 5 represents the time series pattern of the hourly electricity load for each of the 6 industrial consumers. Procedures to improve data collection were here enforced to treat missing data, outliers, typos, seasonality and errors. Week seasonality was observed in all time series (of lag 168). The available data set comprises three years of hourly electricity load of Duke Energy, a production company that operates in Brazil.

[INSERT FIG. 5 HERE]

One problem that comes up when dealing with multiple time series is usually related to the way the time series are related. In this regard it is worth mentioning that for this entire set of electricity load the behavior of the six time series cannot be considered as a multivariate process. The correlation between variables was not significant in any pair of the six time series. Also, some factors such as missing data, different number of samples and different intervals, made this kind of approach less likely to occur. In this way, each series needs to be independently addressed. Some time series that come from industrial customers with the same activity could be explained by the same variables, but this fact was not considered in this paper. Only one multivariate specific method with the same parameters is not usually able to fit the whole set of time series.

Table IV presents the results for the six industrial consumers. The time series characteristics of lag seasonality, mean, standard error of the mean, standard deviation, skewness and kurtosis were computed. Historical data and consumer's information about trading in the spot market was also provided.

[INSERT TABLE IV HERE]

It was previous known that the consumers Delphi and Johnson didn't trade any electricity in the spot market. In this case the detection of change point using the multivariate approach means only false alarm because they didn't present any real change point. Delphi has presented only one change point after 1800 points mainly due to the complexity component of the Hjorth descriptor. This result is not significant considering that the nonlinear time series contains the highest volatility among the six time series. In a traditional control chart this result would be around 370 points.

For the industrial consumers Cenu, Food-Town, Oxicap and Globo the number of points to detect the change point was computed and the results were inferior to the lag seasonality. The seasonality was computed using a Fast Fourier Transform (FFT) algorithm. The Hjorth descriptor's components were obtained but a pattern was not found for the six time series. The number of points related to the change point detection is averaged once the above mentioned time series have several change points in the historical data.

V. CONCLUSIONS AND FURTHER RESEARCH

In this study, a novel multivariate method based on the Hjorth's descriptors of activity, mobility and complexity was applied to detect dynamic changes in nonlinear time series. A multivariate control chart monitoring special causes of variation on the normalized descriptors was applied to a change point detection problem on six nonlinear time series. The simulated results have shown that the method was able to detect the change point adequately and with a satisfactory average run length.

Related to the case study, it is correct to conclude that the proposed approach was quite competent on detecting the change point of the short term electricity load for the six industrial consumers.

As for further step the authors are developing a pattern recognition algorithm to an online control chart monitoring. In this new study several control chart run tests will be detected in real time using neural networks. This mechanism is necessary for change point detection of several time series like EEG signals and machining process.

Another further research is the evaluation of the present approach in face of the Kolmogorov-Sinai entropy.

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