MEASUREMENT SYSTEM ANALYSIS APPLIED TO AISI 12L14 STEEL TURNING

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ABSTRACT

The aim of this paper is to explore how measurement systems having correlated characteristics are analyzed through studies of gage repeatability and reproducibility (GR&R). Moreover, to increase the reliability of the results, the multivariate evaluation index was estimated using four weighting strategies based on arithmetic and geometric means. This GR&R study considered 12 parts, 3 operators, 4 replicates, and 5 responses (R_a, R_y, R_z, R_q and R_t). The data set has a correlation structure that determines 84.5% of explanation for the first pair of eigenvalues extracted from total variation matrix. The weighted indexes are compared with univariate and multivariate indexes in the literature. One of the more significant findings to emerge from this study is that the univariate approach is unable to provide a single assessment to the multivariate measurement system. It was also shown that the weighted strategies in this paper were more accurate in estimating the evaluation index for multivariate GR&R studies.

Keywords: *measurement system analysis, repeatability and reproducibility, multivariate analysis of variance, roughness*

1. Introduction

Quality improvement projects are often characterized by their objective to reduce variability and achieve zero-defect production. If a product fails to conform to these standards, analysts generally blame the process and then act to improve process capability. In some instances, however, the process capability may be fine. Yet the measurement error, when compared to the variability of the process, remains unacceptable (Al-Refaie and Bata, 2010). Hence, before a team of analysts tries to improve a process, they should investigate both the variability of the measurement process as well as the variability of the manufacturing process.

To draw inferences regarding products and process quality, manufacturers use quantitative methods. Such methods (e.g., process capability indexes and control charts) incorporate data into the decision-making process. Prior to obtaining data, a manufacturer should help ensure its validity by evaluating the measuring device (Majeske, 2008). According to Wu et al. (2009), the inevitable variations in process measurements arise from two sources: the manufacturing process and the gage. In manufacturing, a measurement system is not used to produce an exact dimension of a part. Such a system provides measurements that, due to errors (random and systematic), vary from the true value (AIAG, 2010). In any activity involving measurements, whatever observed variability not due to the product/process itself, σ_P^2, is due to measurement error or variability in the measurement system, σ_{MS}^2 (Costa et al., 2005; Li and Al-Refaie, 2008; Senol, 2004; Woodall and Borror, 2008).

To identify the components of variations in the precision and accuracy assessments of measuring instruments, researchers often rely on measurement system analysis (MSA). The purposes of MSA are to: (1) determine the extent of the observed variability caused by a test instrument, (2) identify the sources of variability in a testing system, and (3) assess the capability of a test instrument (Burdick et al., 2003). According to He et al. (2011), MSA is an important element of Six Sigma as well as of the ISO/TS 16949 standards. Gage Repeatability and Reproducibility (GR&R) is the most common study in MSA to evaluate statistical variations in the measurement process. Repeatability is the variation in measurements obtained by one measuring instrument when used several times by one appraiser while measuring an identical characteristic on the same part. Reproducibility is the variation in the average of measurements made by different appraisers using the same gage when measuring a characteristic on one part (Burdick et al., 2003; Erdmann et al., 2010; Wu et al., 2009). GR&R aims to determine that a measurement system's variability is less than that of the monitoring process (Al-Refaie and Bata, 2010; Wang and Chien, 2010).

As emphasized above, in order to analyze the process capability of a quality improvement project, a team of analysts should evaluate first the capability of the measurement system. Two methods commonly used in the analysis of a GR&R study are: (1) an analysis of variance (ANOVA) approach followed by estimation of the appropriate variance components; and (2) an X-bar and Range chart that estimates the standard deviations of the components of gage variability (Wang and Chien, 2010). Analysts prefer the ANOVA method because it measures the operator-to-part interaction gage error; this variation is not included in the X-bar and Range method (AIAG, 2010). Burdick et al. (2003) provided a good review of methods for conducting and analyzing measurement system capability studies, which are based on the analysis of variance approach. Kaija et al. (2010) used the Six Sigma DMAIC (define, measure, analyze, improve, control) approach to evaluate a process of printing a dielectric layer with an inkjet printer. Initially, a GR&R study was conducted to evaluate the proportion of variation caused by the measurement system and process variation. Experiments were then planned and analyzed to identify the parameters having the most significant effects on the output variables of the dielectric layer's insulating layer and surface roughness. Li and Al-Refaie (2008) used the Six Sigma DMAIC procedure to improve quality through enhancing the measuring system capability of the wood industry. The measurement system assessed through GR&R had been considered unacceptable. To improve it, analysts implemented corrective actions, including operator training, proper selection of measuring instruments, and improved measuring procedures. In a second GR&R study, the authors concluded that the corrective actions had reduced the *%R&R* index (percentage of repeatability and reproducibility) by 39.38% and had improved the *ndc* index (number of distinct categories) by 168.84%.

Other studies have employed GR&R to evaluate measurement systems. In proposing a procedure to evaluate measurement systems and process capabilities, Al-Refaie and Bata (2010) used GR&R along with four quality measures. The quality measures were: precision-to-tolerance ratio (*P/T*), signal-to-noise ratio (*SNR*), discrimination ratio (*DR*), and process capability index (*Cp* or *Cpk*). Costa et al. (2005) addressed the design and implementation of a measurement system that permitted the evaluation—quantitatively, objectively, and systematically—of the superficial paper waviness in industrial practice. The process of designing the measurement system was presented considering all its stages, from selection and evaluation of the measuring device (using GR&R) to the generation and validation of the statistical model of measurement. Senol (2004) used an experimental design including laboratory factors as a measurement variability factor in MSA studies. This study concluded that environmental and atmospheric conditions, often overlooked in GR&R studies, might represent a significant contribution to the variability in measurements.

The bulk of the studies associated with analyzing the quality and efficiency of measurement systems are so far limited to a discussion of one single critical-to-quality characteristic (CTQ). Currently, the ANOVA method for GR&R studies can be applied only to univariate data (Wang and Yang, 2007). In assessing measurement systems that measure multiple characteristics, the analyst must consider the correlation structure of the CTQs, a task more suited to multivariate methods. Flynn et al. (2009) used regression analysis to analyze the comparative performance capability between two functionally equivalent but technologically different automatic measurement systems. The systems were used for acceptance testing of a unit under test. For such accurate measurements as repeatability and reproducibility, the "pass/fail" criteria for a test unit were inappropriate. Hence, the authors proposed a methodology that used principal component analysis (PCA) and multivariate analysis of variance (MANOVA) to examine whether there was a statistically significant difference between the system's measurements. He et al. (2011) proposed an online multivariate MSA approach to detecting faulty test instruments in a multisite testing system. The multivariate data were transformed using PCA. The values of the principal components of each test instrument were then compared with the control limits obtained by analyzing the principal components of all test instruments. Majeske (2008) used the MANOVA method to estimate the variance-covariance matrix for GR&R studies with one, two, and three significant factors. This work evaluated a measurement system using data from a GR&R study of a sheet-metal body panel. In so doing it demonstrated how to adjust a MANOVA model and estimate multivariate criteria (P/T_m, $\%R\&R_m$ and SNR_m). Wang and Chien (2010) used the process-oriented basis representation method (POBREP) to evaluate a measurement process with multivariate data. The results showed that POBREP outperformed other methods such as PCA and ANOVA. The POBREP was able to identify specific causes of production problems and map those into a basis matrix. Wang and Yang (2007) presented a GR&R study with multiple characteristics using the PCA method. To assess the adequacy of the measurement system, the study employed two composite indexes: precision-to-tolerance ratio and measurement-variation-to-total-variation-of-measurement-system ratio. The case study showed that, for estimating the indexes, PCA outperformed ANOVA.

The main objective of this paper is to present a multivariate analysis of a measurement system through studies of repeatability and reproducibility of the measurement process. Additionally, four multivariate indexes using different weighting approaches were adopted to achieve more accurate results. The study evaluated measurements of roughness parameters, obtained from AISI 12L14 steel turning machined with carbide tools. In this GR&R study, the following are considered: 12 parts, 3 operators, 4 replicates, and 5 responses (R_a, R_y, R_z, R_q and R_t) with a correlation structure that determined 84.5% of explanation to the first pair of eigenvalues extracted from the total variation matrix. The results show that ANOVA is not a suitable means of treating multiple responses with significant correlations. MANOVA method provided a single evaluation; however, *G* index seems to misestimate the multivariate evaluation index when it is compared to univariate *%R&R* indexes. Weighted indexes adopted in this article obtained more coherent results in assessing multivariate GR&R studies.

The remainder of this paper is structured as follows. Section 2 shows how to evaluate a measurement system using univariate method by ANOVA. Section 3 details the MANOVA method and the four multivariate indexes adopted in this article. Section 4 presents GR&R studies applied to the roughness parameters of AISI 12L14 steel turning; the data is evaluated using univariate and multivariate methods. Finally, Section 5 presents the main findings involving the analysis using univariate and multivariate methods for multivariate GR&R studies.

2. GR&R study using ANOVA

In many processes involving measurements of manufactured products for a single CTQ, the variability may be due to a measurement error, to variability in the measuring device, or to variability in the product/process itself. A complete model for a GR&R study with *p* parts, *o* operators, and *r* replicates is made up of two-factor crossed design with interaction according to Eq. (1) (Al-Refaie and Bata, 2010; Burdick et al., 2003; Deldossi and Zappa, 2011; Erdmann et al., 2010):

$$\mathbf{ctq} = \mu + \alpha_i + \beta_j + (\alpha\beta)_{ij} + \varepsilon_{ijk} \begin{cases} i = 1,2,\ldots,p \\ j = 1,2,\ldots,o \\ k = 1,2,\ldots,r \end{cases} \tag{1}$$

where **ctq** is the response variable measured; μ is the mean of the measured values; $\alpha_i \sim N(0,\sigma_\alpha)$, $\beta_j \sim N(0,\sigma_\beta)$, $\alpha\beta_{ij} \sim N(0,\sigma_{\alpha\beta})$ and $\varepsilon_{ijk} \sim N(0,\sigma_\varepsilon)$ are random variables statistically independent part, operator, interaction and the error term, respectively. It is possible to translate the above components of variance into notation GR&R to (Kaija et al., 2010; Li and Al-Refaie, 2008; Senol, 2004; White and Borror, 2011):

$$\begin{aligned}&\sigma_P^2 = \sigma_\alpha^2,\\ &\sigma_{repeatability}^2 = \sigma_\varepsilon^2,\ \sigma_{reproducibility}^2 = \sigma_\beta^2 + \sigma_{\alpha\beta}^2\\ &\sigma_{MS}^2 = \sigma_{repeatability}^2 + \sigma_{reproducibility}^2,\\ &\sigma_T^2 = \sigma_P^2 + \sigma_{MS}^2\end{aligned} \tag{2}$$

The variance components of the model (1) in Eq (2) can be estimated using the method Analysis of Variance (ANOVA), according to Eqs. (3) – (7).

$$\hat{\sigma}_P^2 = \hat{\sigma}_\alpha^2 = \frac{MSP - MSPO}{or} \tag{3}$$

$$\begin{aligned}\hat{\sigma}_{reproducibility}^2 &= \hat{\sigma}_\beta^2 + \hat{\sigma}_{\alpha\beta}^2\\ &= \frac{MSO - MSPO}{pr} + \frac{MSPO - MSE}{r}\end{aligned} \tag{4}$$

$$\hat{\sigma}_{repeatability}^2 = \hat{\sigma}_\varepsilon^2 = MSE \tag{5}$$

$$\hat{\sigma}_{MS}^2 = \hat{\sigma}_{repeatability}^2 + \hat{\sigma}_{reproducibility}^2 \tag{6}$$

$$\hat{\sigma}_T^2 = \hat{\sigma}_P^2 + \hat{\sigma}_{MS}^2 \tag{7}$$

where *MSP*, *MSO*, *MSPO*, and *MSE* are, respectively, the mean squares for the factors part, operator, interaction, and the error term. Also, if the interaction effect is not significant, the complete model can be reduced to:

$$\mathbf{ctq} = \mu + \alpha_i + \beta_j + \varepsilon_{ijk} \tag{8}$$

Now the components of variance for parts (process) and reproducibility (operators) in Model (8) are estimated by:

$$\hat{\sigma}_P^2 = \hat{\sigma}_\alpha^2 = \frac{MSP - MSE}{or} \tag{9}$$

$$\hat{\sigma}_{reproducibility}^2 = \hat{\sigma}_\beta^2 = \frac{MSO - MSE}{pr} \tag{10}$$

A common index in GR&R studies is used to determine the acceptance of a measurement system. The AIAG (2010) recommended evaluating a measurement system by scaling the standard deviation of measurement error to the total standard deviation of the observed process. This statistic, called the percentage of R&R, is defined as:

$$\%R\&R = \left(\frac{\sigma_{MS}}{\sigma_T}\right)100\% \tag{11}$$

If the measurement system, according to the index, is less than 10%, then it is considered acceptable. If it is between 10% and 30%, then it is considered marginal—acceptable depending on the application, the cost of the measurement device, the cost of repair and other factors. If, according to the index, the measurement system exceeds 30%, then it is considered unacceptable and should be improved (AIAG, 2010; Al-Refaie and Bata, 2010; Woodall and Borror, 2008).

3. GR&R study using MANOVA

When a GR&R study considers a two-factors cross design with interaction for multiple CTQs (*q* characteristics), the model is given by (He et al., 2011; Majeske, 2008; Wang and Chien, 2010):

$$\begin{aligned}\mathbf{CTQ} &= \begin{bmatrix} CTQ_{11} & CTQ_{12} & \cdots & CTQ_{1q}\\ CTQ_{21} & CTQ_{22} & \cdots & CTQ_{2q}\\ \vdots & \vdots & \ddots & \vdots\\ CTQ_{n1} & CTQ_{n2} & \cdots & CTQ_{nq}\end{bmatrix}\\ &= \boldsymbol{\mu} + \boldsymbol{\alpha}_\mathbf{i} + \boldsymbol{\beta}_\mathbf{j} + (\boldsymbol{\alpha\beta})_\mathbf{ij} + \boldsymbol{\varepsilon}_\mathbf{ijk}\end{aligned} \tag{12}$$

where $\mathbf{CTQ} = (\mathbf{CTQ_1}, \mathbf{CTQ_2}, \ldots, \mathbf{CTQ_q})$ and $\boldsymbol{\mu} = (\boldsymbol{\mu_1}, \boldsymbol{\mu_2}, \ldots, \boldsymbol{\mu_q})$ are constant vectors; $\boldsymbol{\alpha}_\mathbf{i} \sim N(\mathbf{0}, \boldsymbol{\Sigma}_\alpha)$, $\boldsymbol{\beta}_\mathbf{j} \sim N(\mathbf{0}, \boldsymbol{\Sigma}_\beta)$, $\boldsymbol{\alpha\beta}_\mathbf{ij} \sim N(\mathbf{0}, \boldsymbol{\Sigma}_{\alpha\beta})$, and $\boldsymbol{\varepsilon}_\mathbf{ijk} \sim N(\mathbf{0}, \boldsymbol{\Sigma}_\varepsilon)$ are random vectors statistically independent of each other. It is possible to translate the above components of variance into notation GR&R to:

$$\begin{aligned}&\hat{\boldsymbol{\Sigma}}_\mathbf{P} = \hat{\boldsymbol{\Sigma}}_{\boldsymbol{\alpha}},\\ &\hat{\boldsymbol{\Sigma}}_\mathbf{repeatability} = \hat{\boldsymbol{\Sigma}}_{\boldsymbol{\varepsilon}},\quad \hat{\boldsymbol{\Sigma}}_\mathbf{reproducibility} = \hat{\boldsymbol{\Sigma}}_{\boldsymbol{\beta}} + \hat{\boldsymbol{\Sigma}}_{\boldsymbol{\alpha\beta}}\\ &\hat{\boldsymbol{\Sigma}}_\mathbf{MS} = \hat{\boldsymbol{\Sigma}}_\mathbf{repeatability} + \hat{\boldsymbol{\Sigma}}_\mathbf{reproducibility},\\ &\hat{\boldsymbol{\Sigma}}_\mathbf{T} = \hat{\boldsymbol{\Sigma}}_\mathbf{P} + \hat{\boldsymbol{\Sigma}}_\mathbf{MS}\end{aligned} \tag{13}$$

The variance components of the model (12) in Eq. (13) can be estimated using the method of Multivariate Analysis of Variance (MANOVA). Before estimating the variance-covariance matrices, $\boldsymbol{\Sigma}_\mathbf{P}$, $\boldsymbol{\Sigma}_\mathbf{MS}$ and $\boldsymbol{\Sigma}_\mathbf{T}$ are calculated mean squares matrices for part (**MSP**), operator (**MSO**), part*operator interaction (**MSPO**) and the error term (**MSE**) as such:

$$\mathbf{MSP_{ab}} = \frac{or}{p-1}\sum_{i=1}^{p}\left(\overline{m}_{a_{i..}} - \overline{m}_{a_{...}}\right)\left(\overline{m}_{b_{i..}} - \overline{m}_{b_{...}}\right) \tag{14}$$

$$\mathbf{MSO_{ab}} = \frac{pr}{(o-1)}\sum_{j=1}^{o}\left(\overline{m}_{a_{.j.}} - \overline{m}_{a_{...}}\right)\left(\overline{m}_{b_{.j.}} - \overline{m}_{b_{...}}\right) \tag{15}$$

$$\begin{aligned}\mathbf{MSPO_{ab}} = \frac{r}{(p-1)(o-1)} &\sum_{i=1}^{p}\sum_{j=1}^{o}\left(\overline{m}_{a_{ij.}} - \overline{m}_{a_{i..}} - \overline{m}_{a_{.j.}} + \overline{m}_{a_{...}}\right)\\ &\left(\overline{m}_{b_{ij.}} - \overline{m}_{b_{i..}} - \overline{m}_{b_{.j.}} + \overline{m}_{b_{...}}\right)\end{aligned} \tag{16}$$

$$\mathbf{MSE_{ab}} = \frac{1}{po(r-1)}\sum_{i=1}^{p}\sum_{j=1}^{o}\sum_{k=1}^{r}\left(\overline{m}_{a_{ijk}} - \overline{m}_{a_{...}}\right)\left(\overline{m}_{b_{ijk}} - \overline{m}_{b_{...}}\right) \tag{17}$$

After that, variance-covariance matrices are estimated for process ($\hat{\boldsymbol{\Sigma}}_\mathbf{P}$), reproducibility ($\hat{\boldsymbol{\Sigma}}_\mathbf{reproducibility}$), repeatability ($\hat{\boldsymbol{\Sigma}}_\mathbf{repeatability}$), measurement system ($\hat{\boldsymbol{\Sigma}}_\mathbf{MS}$) and total variation ($\hat{\boldsymbol{\Sigma}}_\mathbf{T}$), according to Eqs. (18) – (22):

$$\hat{\boldsymbol{\Sigma}}_{\mathbf{P}} = \hat{\boldsymbol{\Sigma}}_{\boldsymbol{\alpha}} = \frac{\mathbf{MSP} - \mathbf{MSPO}}{or} \tag{18}$$

$$\hat{\boldsymbol{\Sigma}}_{\mathbf{reproducibility}} = \hat{\boldsymbol{\Sigma}}_{\boldsymbol{\beta}} + \hat{\boldsymbol{\Sigma}}_{\boldsymbol{\alpha\beta}} = \frac{\mathbf{MSO} - \mathbf{MSPO}}{pr} + \frac{\mathbf{MSPO} - \mathbf{MSE}}{r} \tag{19}$$

$$\hat{\boldsymbol{\Sigma}}_{\mathbf{repeatability}} = \hat{\boldsymbol{\Sigma}}_{\boldsymbol{\varepsilon}} = \mathbf{MSE} \tag{20}$$

$$\hat{\boldsymbol{\Sigma}}_{\mathbf{MS}} = \hat{\boldsymbol{\Sigma}}_{\mathbf{repeatability}} + \hat{\boldsymbol{\Sigma}}_{\mathbf{reproducibility}} \tag{21}$$

$$\hat{\boldsymbol{\Sigma}}_{\mathbf{T}} = \hat{\boldsymbol{\Sigma}}_{\mathbf{P}} + \hat{\boldsymbol{\Sigma}}_{\mathbf{MS}} \tag{22}$$

If the interaction effect is not significant, the model of Eq. (12) can be reduced to the model of Eq. (23). In this case, the mean square matrix of **MSE** is estimated by Eq. (24) and variance-covariance matrices for part (process) and reproducibility (operator) are estimated using Eqs. (25) and (26). **MSP**, **MSO**, $\hat{\boldsymbol{\Sigma}}_{\mathbf{Repeatabilty}}$, $\hat{\boldsymbol{\Sigma}}_{\mathbf{MS}}$ and $\hat{\boldsymbol{\Sigma}}_{\mathbf{T}}$ are estimated using previously mentioned Eqs. (14), (15), (20), (21) and (22), respectively.

$$\mathbf{CTQ} = \boldsymbol{\mu} + \boldsymbol{\alpha}_{\mathbf{i}} + \boldsymbol{\beta}_{\mathbf{j}} + \boldsymbol{\varepsilon}_{\mathbf{ijk}} \begin{cases} i = 1,2,\ldots,p \\ j = 1,2,\ldots,o \\ k = 1,2,\ldots,r \end{cases} \tag{23}$$

$$\mathbf{MSE}_{\mathbf{ab}} = \frac{1}{por - p - o + 1} \sum_{i=1}^{p}\sum_{j=1}^{o}\sum \left(\overline{m}_{a_{ijk}} - \overline{m}_{a_{i..}} - \overline{m}_{a_{.j.}} + \overline{m}_{a_{...}}\right)\left(\overline{m}_{b_{ijk}} - \overline{m}_{b_{i..}} - \overline{m}_{b_{.j.}} + \overline{m}_{b_{...}}\right) \tag{24}$$

$$\hat{\boldsymbol{\Sigma}}_{\mathbf{P}} = \hat{\boldsymbol{\Sigma}}_{\boldsymbol{\alpha}} = \frac{\mathbf{MSP} - \mathbf{MSE}}{or} \tag{25}$$

$$\hat{\boldsymbol{\Sigma}}_{\mathbf{reproducibility}} = \hat{\boldsymbol{\Sigma}}_{\boldsymbol{\beta}} = \frac{\mathbf{MSO} - \mathbf{MSE}}{pr} \tag{26}$$

The multivariate version of the *%R&R* index proposed by Majeske (2008) is called here *G* index and is calculated by Eq. (27). λ_{MS_i} and λ_{T_i} $\forall i = 1,2,\ldots,q$ are eigenvalues extracted from variance-covariance matrices, $\boldsymbol{\Sigma}_{\mathbf{MS}}$ and $\boldsymbol{\Sigma}_{\mathbf{T}}$.

$$G = \left(\prod_{i=1}^{q}\sqrt{\frac{\boldsymbol{\lambda}_{\mathbf{MS_i}}}{\boldsymbol{\lambda}_{\mathbf{T_i}}}}\right)^{1/q} 100\% \tag{27}$$

To obtain the evaluation index to the measurement system, Majeske (2008) applied geometric mean on the ratio $\sqrt{\lambda_{MS}/\lambda_T}$. This strategy does not determine greater importance to the most significant pair of eigenvalues, extracted from variance-covariance matrices. As a result, this article adopts a weighted approach upon $\sqrt{\lambda_{MS}/\lambda_T}$ ratio to propose four new evaluation indexes for multivariate measurement systems. These new indexes, WA_T, WA_{MS}, WG_T and WG_{MS}, can be obtained based on Eqs. (28) and (29).

$$WA = \sum_{i=1}^{q}\left(W_i\sqrt{\frac{\lambda_{SM_i}}{\lambda_{T_i}}}\right) 100\% \tag{28}$$

$$WG = \prod_{i=1}^{q}\left(\sqrt{\frac{\lambda_{SM_i}}{\lambda_{T_i}}}\right)^{W_i} 100\% \tag{29}$$

where λ_{SM} and λ_T are eigenvalues extracted from variance-covariance matrices $\hat{\boldsymbol{\Sigma}}_{\mathbf{MS}}$ and $\hat{\boldsymbol{\Sigma}}_{\mathbf{T}}$, respectively; $W_i \; \forall i = 1,\ldots,q$ are the explanation percentage of the eigenvalues extracted from either $\hat{\boldsymbol{\Sigma}}_{\mathbf{T}} : W_i = \left(\lambda_{T_i} \big/ \sum_{j=1}^{q}\lambda_{T_j}\right)$ or $\hat{\boldsymbol{\Sigma}}_{\mathbf{MS}} : W_i = \left(\lambda_{SM_i} \big/ \sum_{j=1}^{q}\lambda_{SM_j}\right)$. The WA_T and WA_{MS} indexes are obtained by calculating the weighted arithmetic mean according to Eq. (28). The first index, WA_T, weights the $\sqrt{\lambda_{MS}/\lambda_T}$ ratio using the explanation percentage of the eigenvalues extracted from total variation matrix. The second index, WA_{MS}, weights the $\sqrt{\lambda_{MS}/\lambda_T}$ ratio through the explanation percentage of the eigenvalues extracted from measurement system matrix. On the other hand, the WG_T and WG_{MS} indexes are calculated using weighted geometric mean in the Eq. (29). The first index, WG_T, weights the $\sqrt{\lambda_{MS}/\lambda_T}$ ratio using the explanation percentage of the eigenvalues extracted from total variation matrix. The second index, WG_{MS}, weights the $\sqrt{\lambda_{MS}/\lambda_T}$ ratio through the explanation percentage of the eigenvalues extracted from measurement system matrix. The acceptance criterion of the measurement system is the same as described in section 2 (Majeske, 2008).

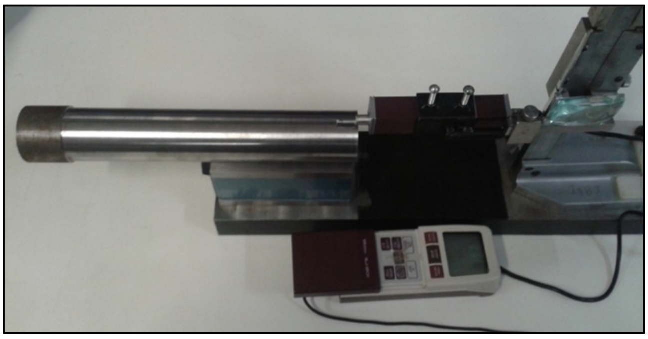
4. Numerical Example

In this multivariate GR&R study, five roughness parameters are analyzed: $\mathbf{R_a}$ (arithmetic average), $\mathbf{R_y}$ (maximum), $\mathbf{R_z}$ (ten point height), $\mathbf{R_q}$ (root mean square), and $\mathbf{R_t}$ (maximum peak to valley). The work piece, machined on a CNC lathe (see Fig. 1a), was AISI 12L14 steel (0.090% C, 0.030% Si, 1.240% Mn, 0.046% P, 0.273% S, 0.150% Cr, 0.080% Ni, 0.260% Cu, 0.001% Al, 0.020% Mo , 0.280% Pb, 0.0079% N2). The machining parameters used in this study were cutting speed of *345 m min*$^{-1}$, feed rate of *0.086 mm rev*$^{-1}$, and depth of cut of *0.680 mm*. Carbide inserts were used of ISO P35 class, coated with three toppings (Ti (C, N), Al_2O_3, TiN), (GC Sandvik 4035) geometry ISO SNMG 09 03 04 – PM, and tool holder ISO DSBNL 1616H 09.

Adopting these experimental conditions, four parts were machined and three noise conditions were considered: slenderness of the part, measuring position, and tool wear. The slenderness (*S*) relates the diameter (*D*) with the length (*L*) of the part, according to the relation *S=L/D*. The parts were classified as slender and non-slender, for the same part length, with *D=30 mm* and *D=50 mm*, respectively. The regions of measurement were adopted: close to the main spindle, center and close to the barrel. The tool wear noise considered new tool and worn tool which it was measured on the edge of approximately *0.3 mm*.

Using the experimental design and considering the noise conditions mentioned above, the GR&R study adopted *p=12 parts*, *o=3* operators and *r=4* replicates. Table 1 contains the data for the GR&R study. The device evaluated in this study was a portable roughness meter set to a cut-off length of 0.25 shown in Fig. 1b.

(a)



(b)

Fig. 1. (a) AISI 12L14 steel machined on a CNC lathe with a 5.5 kW spindle motor with conventional roller bearings; **(b)** Mitutoyo portable roughness meter model Surftest SJ-201P.

4.1 Univariate GR&R study based on ANOVA

A strategy commonly used in quality improvement projects is to prioritize the quality characteristic. For this study, the parameter $\mathbf{R_a}$, widely used in most manufacturing processes, was selected to evaluate the measurement system. To run the univariate GR&R study, a two-way analysis of variance with interaction, Eq. (1), was adjusted to $\mathbf{R_a}$. The interaction term was not significant for a significance level of 0.05, so the model could be adjusted to a reduced model without the interaction term in Eq. (8). Using Eqs. (5)-(7), (9), and (10), the study calculated variances for parts (process), repeatability, reproducibility (operators), measurement system, and total variance. Table 2 shows the square roots of theses variances and, with Eq. (11), the *%R&R* index. The index *%R&R =18.22%* classifies the measurement system as marginal, having potential for improvement.

However, the prioritization of the CTQs is not satisfactory in evaluating the surface roughness of a machined part. The parameter $\mathbf{R_a}$ alone is insufficient for describing a surface completely. Its disadvantage is evident when a non-typical peak or valley is detected on the surface. Nevertheless, it does not interfere in the calculation of the average value, keeping the defect nearly concealed. To evaluate $\mathbf{R_a}$, the analyst may be interested in a complementary parameter. The parameter $\mathbf{R_y}$ has wide acceptance and can be a good choice because it provides information about the deterioration of the vertical surface part. To run the univariate GR&R study, a two-way analysis of variance with interaction, Eq. (1), was adjusted to $\mathbf{R_y}$. The interaction term was not significant for a significance level of 0.05, so the model could be adjusted to a reduced model without the interaction term. Then, variances in Eq. (2) were estimated and their square roots appear in Table 2 as $\hat{\sigma}_P$, $\hat{\sigma}_{MS}$, and $\hat{\sigma}_T$. Finally, Eqs. (11) was used to estimate the *%R&R* index. The index *%R&R = 38.18%* classified the measurement system as unacceptable.

Individually, the parameter $\mathbf{R_y}$ also fails to provide sufficient information about the surface. Indeed, various forms of surface roughness may have the same value $\mathbf{R_y}$. When $\mathbf{R_a}$ and $\mathbf{R_y}$ (as a supplement to $\mathbf{R_a}$) are evaluated using univariate statistical techniques, the analyst cannot verify the measurement system's acceptability. Since the same measurement device measures all roughness parameters, a more detailed analysis was able to show that these responses were highly correlated (see Table 3). Therefore, assessing the measurement system considering independent responses may not be the most appropriate method. It was decided to display the parameters of surface roughness in a vector and use a multivariate approach to evaluate the measurement system.

4.2 Multivariate GR&R study based on MANOVA

Before beginning the measurement system analysis using MANOVA, the data in Table 1 were standardized by subtracting the mean and dividing by the standard deviation for each observation. The standardization of data is important not only when the variables are in different units but also when the variables are at different scales. Thus, to perform this multivariate GR&R study, the analyst adjusted the standardized data of Table 1 using a two-way multivariate analysis of variance according to the model in Eq. (12). The interaction term was not significant for a significance level of 0.05. Thus, the model could be adjusted to a reduced model without the interaction term, according to Eq. (23). Then, using Eqs. (14), (15), and (24), the matrices of the mean squares for the factor part, the factor operator, and the error term were estimated. The matrices are:

$$\mathbf{MSP} = \begin{pmatrix} 12.5745 & 7.5039 & 10.1185 & 12.3004 & 7.2875 \\ 7.5039 & 11.1506 & 10.5769 & 8.2761 & 11.2526 \\ 10.1185 & 10.5769 & 11.8887 & 10.7970 & 10.5188 \\ 12.3004 & 8.2761 & 10.7970 & 12.2838 & 8.0766 \\ 7.2875 & 11.2526 & 10.5188 & 8.0766 & 11.4015 \end{pmatrix} \quad (30)$$

$$\mathbf{MSO} = \begin{pmatrix} 0.0070 & -0.0079 & 0.0090 & 0.0092 & 0.0063 \\ -0.0079 & 0.0088 & -0.0100 & -0.0104 & -0.0072 \\ 0.0090 & -0.0100 & 0.0117 & 0.0116 & 0.0069 \\ 0.0092 & -0.0104 & 0.0116 & 0.0123 & 0.0092 \\ 0.0063 & -0.0072 & 0.0069 & 0.0092 & 0.0109 \end{pmatrix} \quad (31)$$

$$\mathbf{MSE} = \begin{pmatrix} 0.0359 & 0.0479 & 0.0447 & 0.0424 & 0.0447 \\ 0.0479 & 0.1564 & 0.1017 & 0.0715 & 0.1353 \\ 0.0447 & 0.1017 & 0.0939 & 0.0664 & 0.0910 \\ 0.0424 & 0.0715 & 0.0664 & 0.0604 & 0.0658 \\ 0.0447 & 0.1353 & 0.0910 & 0.0658 & 0.1351 \end{pmatrix} \quad (32)$$

Table 1

Observed measurements of roughness parameters for the multivariate GR&R study.

		j=1					j=2					j=3				
i	**k**	$\mathbf{R_z}$	$\mathbf{R_y}$	$\mathbf{R_t}$	$\mathbf{R_q}$	$\mathbf{R_a}$	$\mathbf{R_z}$	$\mathbf{R_y}$	$\mathbf{R_t}$	$\mathbf{R_q}$	$\mathbf{R_a}$	$\mathbf{R_z}$	$\mathbf{R_y}$	$\mathbf{R_t}$	$\mathbf{R_q}$	$\mathbf{R_a}$
1	1	6.34	7.73	8.81	1.67	1.39	6.37	8.29	9.25	1.67	1.38	6.39	8.38	9.25	1.67	1.38
2	1	7.61	9.19	9.46	1.85	1.54	7.57	9.17	9.43	1.84	1.53	7.60	9.18	9.41	1.84	1.53
3	1	6.43	7.67	7.81	1.50	1.23	6.11	7.78	8.46	1.55	1.27	6.44	7.69	7.83	1.50	1.23
4	1	7.52	9.01	9.01	2.10	1.84	7.53	8.86	8.94	2.10	1.84	7.54	9.03	9.03	2.10	1.84
5	1	7.28	7.77	8.04	2.30	2.07	7.26	7.84	8.02	2.27	2.05	7.26	7.84	8.02	2.27	2.05
6	1	7.34	7.92	7.94	2.35	2.12	7.33	7.92	7.95	2.37	2.14	7.32	7.94	7.96	2.37	2.14
7	1	9.24	10.55	10.55	2.39	1.93	9.27	10.55	10.55	2.39	1.92	9.22	10.54	10.54	2.38	1.91
8	1	3.74	4.04	4.04	0.93	0.75	3.95	4.32	4.32	0.96	0.77	3.88	4.32	4.32	0.97	0.78
9	1	4.17	4.84	5.02	0.98	0.79	4.27	4.87	5.02	1.02	0.81	4.24	4.89	5.05	0.99	0.79
10	1	7.70	8.62	9.25	2.04	1.70	7.83	8.67	9.16	2.10	1.76	7.79	8.66	9.11	2.10	1.76
11	1	6.85	7.51	7.51	2.01	1.73	6.83	7.49	7.53	2.00	1.72	6.83	7.46	7.54	2.00	1.72
12	1	7.45	8.29	8.29	2.21	1.95	7.47	8.24	8.24	2.21	1.95	7.46	8.22	8.22	2.21	1.94
1	2	6.63	7.56	7.79	1.80	1.52	6.64	7.57	7.78	1.80	1.52	6.63	7.58	7.74	1.80	1.52
2	2	6.46	8.75	8.85	1.58	1.20	6.03	9.55	9.55	1.47	1.17	5.96	9.30	9.30	1.46	1.17
3	2	6.81	8.27	8.76	1.65	1.29	6.72	7.85	8.82	1.56	1.19	6.86	8.13	8.77	1.61	1.24
4	2	8.02	9.71	10.12	2.04	1.75	8.10	9.78	10.26	2.05	1.76	8.05	9.70	10.22	2.05	1.76
5	2	7.00	7.43	7.58	2.29	2.08	7.00	7.40	7.61	2.29	2.08	7.01	7.42	7.63	2.29	2.08
6	2	7.31	7.82	7.82	2.38	2.15	7.32	7.78	7.92	2.40	2.18	7.34	7.81	7.96	2.38	2.16
7	2	8.80	10.26	10.26	2.31	1.85	8.72	9.43	9.63	2.34	1.89	8.67	9.64	9.85	2.28	1.82
8	2	4.02	4.77	4.77	1.02	0.82	4.07	4.58	4.58	1.02	0.83	3.91	4.27	4.54	1.00	0.81
9	2	3.92	4.26	4.53	0.99	0.80	3.89	4.26	4.51	0.99	0.80	3.90	4.24	4.49	0.99	0.80
10	2	7.75	9.24	9.91	2.06	1.72	7.79	9.22	9.93	2.07	1.73	7.78	9.23	9.91	2.07	1.72
11	2	6.65	7.23	7.29	2.01	1.73	6.64	7.20	7.39	2.01	1.74	6.64	7.19	7.30	2.01	1.73
12	2	7.53	8.48	8.48	2.17	1.88	7.46	8.47	8.47	2.17	1.89	7.45	8.21	8.25	2.17	1.88
1	3	6.05	7.03	7.07	1.52	1.26	6.06	7.10	7.10	1.51	1.26	6.05	7.00	7.12	1.51	1.26
2	3	5.70	7.60	7.60	1.37	1.11	5.67	7.51	7.51	1.39	1.12	5.67	7.56	7.56	1.37	1.10
3	3	6.51	8.46	8.58	1.65	1.37	5.57	9.18	9.18	1.33	1.03	6.41	8.94	8.94	1.70	1.43
4	3	7.59	9.62	9.62	2.00	1.71	7.65	10.07	10.07	2.03	1.73	7.66	9.86	9.86	2.02	1.72
5	3	7.07	7.45	7.65	2.26	2.04	7.09	7.43	7.65	2.26	2.04	7.09	7.46	7.68	2.26	2.03
6	3	7.19	7.89	7.89	2.39	2.17	7.18	7.86	7.86	2.37	2.16	7.15	7.76	7.76	2.37	2.16
7	3	9.46	10.16	10.23	2.43	1.96	9.54	10.42	10.46	2.44	1.97	9.53	10.36	10.40	2.44	1.96
8	3	3.80	3.92	4.04	1.00	0.81	3.79	3.96	4.06	0.96	0.78	3.75	3.92	4.00	0.95	0.77
9	3	3.90	4.00	4.06	1.04	0.83	3.81	3.89	4.03	1.01	0.80	3.78	3.95	4.03	0.99	0.79
10	3	8.35	10.08	11.38	2.10	1.71	8.49	10.06	11.47	2.11	1.72	8.71	10.22	11.82	2.13	1.72
11	3	6.77	7.10	7.22	2.04	1.76	6.76	7.19	7.23	2.03	1.75	6.76	7.15	7.21	2.03	1.75
12	3	7.70	8.02	8.42	2.22	1.93	7.69	8.01	8.40	2.23	1.94	7.70	8.02	8.40	2.23	1.94
1	4	6.39	7.47	7.55	1.64	1.35	6.32	7.47	7.47	1.63	1.34	6.34	7.49	7.49	1.64	1.35
2	4	7.86	9.40	9.77	1.86	1.50	7.85	9.47	9.83	1.86	1.49	7.93	9.70	10.16	1.86	1.49
3	4	7.00	8.79	8.79	1.75	1.42	7.06	8.81	8.92	1.76	1.42	6.94	8.60	8.85	1.75	1.42
4	4	8.07	10.18	10.18	2.18	1.89	8.04	9.49	9.49	2.18	1.89	8.03	9.49	9.56	2.19	1.90
5	4	6.96	7.16	7.30	2.28	2.07	6.97	7.28	7.38	2.28	2.07	6.98	7.29	7.40	2.28	2.07
6	4	7.24	7.56	7.74	2.35	2.12	7.25	7.49	7.75	2.35	2.12	7.25	7.41	7.70	2.36	2.13
7	4	8.10	8.56	8.73	2.18	1.78	8.11	8.59	8.76	2.18	1.78	8.10	8.54	8.69	2.17	1.77
8	4	4.34	5.36	5.36	1.09	0.88	4.15	5.08	5.14	1.06	0.85	4.07	5.14	5.21	1.03	0.83
9	4	4.49	4.74	5.11	1.12	0.90	4.83	5.90	6.42	1.17	0.94	4.78	6.73	6.78	1.18	0.94
10	4	7.36	8.54	8.99	1.98	1.68	7.39	8.71	9.03	1.99	1.68	7.40	8.76	9.06	1.99	1.68
11	4	6.73	7.13	7.28	2.01	1.73	6.69	7.08	7.23	2.00	1.72	6.68	7.07	7.20	1.99	1.71
12	4	7.54	8.09	8.43	2.08	1.79	7.58	8.12	8.29	2.09	1.80	7.56	8.17	8.29	2.11	1.81

The variance-covariance matrices for part, measurement system and the total variation were then estimated, respectively, using Eqs. (25), (21) and (22) (Table 2). Finally, to evaluate the multivariate indexes of the measuring system's acceptability, the eigenvalues of the matrices, $\mathbf{\Sigma_P}$, $\mathbf{\Sigma_{MS}}$ and $\mathbf{\Sigma_T}$, were computed (Table 4). Using Eq. (27) the index *G=44.64%* was the multivariate criterion used to assess the measurement system for the five CTQs $\mathbf{R_a}$, $\mathbf{R_y}$, $\mathbf{R_z}$, $\mathbf{R_q}$, and $\mathbf{R_t}$. The fact that *G> 30%* classifies the measurement system as unacceptable. Nevertheless, *G=44.64%* seems to misestimate the multivariate evaluation index for assessing the measurement system when it is compared to univariate *%R&R* indexes. Therefore, four multivariate indexes were proposed in addition to the *G* index.

Table 2
Roughness meter classification through univariate and multivariate methods.

Source	Univariate					Multivariate				
	R_a	R_y	R_z	R_q	R_t	G	WA_T	WG_T	WA_{MS}	WG_{MS}
P	0.444	1.564	1.383	0.456	1.696	$\begin{pmatrix} 1.045 & 0.621 & 0.839 & 1.021 & 0.604 \\ 0.621 & 0.916 & 0.873 & 0.684 & 0.926 \\ 0.839 & 0.873 & 1.016 & 0.894 & 0.869 \\ 1.021 & 0.684 & 0.894 & 1.019 & 0.668 \\ 0.604 & 0.926 & 0.869 & 0.668 & 0.939 \end{pmatrix}$				
MS	0.082	0.646	0.428	0.111	0.643	$\begin{pmatrix} 0.035 & 0.047 & 0.044 & 0.042 & 0.044 \\ 0.047 & 0.153 & 0.099 & 0.070 & 0.132 \\ 0.044 & 0.099 & 0.092 & 0.065 & 0.089 \\ 0.042 & 0.070 & 0.065 & 0.059 & 0.065 \\ 0.044 & 0.132 & 0.089 & 0.065 & 0.133 \end{pmatrix}$				
T	0.452	1.693	1.448	0.470	1.813	$\begin{pmatrix} 1.080 & 0.668 & 0.883 & 1.063 & 0.647 \\ 0.668 & 1.069 & 0.972 & 0.754 & 1.059 \\ 0.883 & 0.972 & 1.108 & 0.959 & 0.958 \\ 1.063 & 0.754 & 0.959 & 1.078 & 0.732 \\ 0.647 & 1.059 & 0.958 & 0.732 & 1.071 \end{pmatrix}$				
%R&R	18.22%	38.18%	29.52%	23.66%	35.47%	44.64%	29.30%	29.12%	30.92%	30.23%

Table 3
Correlation structure between roughness parameters.

	R_z	R_y	R_t	R_q
R_y	0.920 [a]			
	0.000 [b]			
R_t	0.908	0.988		
	0.000	0.000		
R_q	0.906	0.734	0.708	
	0.000	0.000	0.000	
R_a	0.839	0.652	0.623	0.989
	0.000	0.000	0.000	0.000

[a] Pearson correlation
[b] p-value

Table 4
Eigenvalues and W_i of matrices $\hat{\mathbf{\Sigma}}_\mathbf{P}$, $\hat{\mathbf{\Sigma}}_\mathbf{MS}$ and $\hat{\mathbf{\Sigma}}_\mathbf{T}$.

	λ_1 / W_1[a](%)	λ_2 / W_2 (%)	λ_3 / W_3 (%)	λ_4 / W_4 (%)	λ_5 / W_5 (%)
P	4.195	0.673	0.063	0.002	0.000
	85.0%	13.6%	1.3%	0.0%	0.0%
MS	0.406	0.042	0.013	0.009	0.003
	86.0%	8.8%	2.7%	1.9%	0.6%
T	4.567	0.743	0.079	0.011	0.006
	84.5%	13.7%	1.5%	0.2%	0.1%

[a] $W_i = \left(\lambda_i \big/ \sum_{j=1}^{q} \lambda_j\right) 100\% \quad i = 1,2,\ldots,q$

To obtain the evaluation index to the measurement system, *G* index applied geometric mean on the ratio $\sqrt{\lambda_{MS}/\lambda_T}$. This strategy does not determine greater importance to the most significant pair of eigenvalues, extracted from variance-covariance matrices. As a result, WA_T, WA_{MS}, WG_T and WG_{MS} were calculated using two different weighted approaches upon $\sqrt{\lambda_{MS}/\lambda_T}$ ratio. According to weighted arithmetic mean in Eq. (28) and weighted geometric mean in Eq. (29), the new multivariate indexes are shown here:

$$\begin{aligned} WA_T &= 29.8\times 0.845 + 23.7\times 0.137 + 39.9\times 0.015 \\ &+ 88.8\times 0.002 + 70.7\times 0.001 \\ &= 29.30\% \end{aligned} \tag{33}$$

$$\begin{aligned} WA_{SM} &= 29.8\times 0.860 + 23.7\times 0.088 + 39.9\times 0.027 \\ &+ 88.8\times 0.019 + 70.7\times 0.006 \\ &= 30.92\% \end{aligned} \tag{34}$$

$$\begin{aligned} WG_T &= 29.8^{0.845}\times 23.7^{0.137}\times 39.9^{0.015} \\ &\times 88.8^{0.002}\times 70.7^{0.001} \\ &= 29.12\% \end{aligned} \tag{35}$$

$$\begin{aligned} WG_{SM} &= 29.8^{0.860}\times 23.7^{0.088}\times 39.9^{0.027} \\ &\times 88.8^{0.019}\times 70.7^{0.006} \\ &= 30.23\% \end{aligned} \tag{36}$$

The criterion used in this work to determine if the estimated multivariate index is correct is based on confidence interval for mean calculated from results obtained by ANOVA method. The lower (LCL) and upper (UCL) limits of the confidence intervals are calculated using Eq. (37):

$$CI = \overline{CTQ} \pm t_{N-1,\,\alpha/2}\,\frac{s}{\sqrt{N}} \tag{37}$$

where $\overline{CTQ}$ is the mean of *%R&R* between $\mathbf{R_a}$, $\mathbf{R_y}$, $\mathbf{R_z}$, $\mathbf{R_q}$, and $\mathbf{R_t}$; *s* is the standard deviation; *N* is the sample size and $t_{N-1,\alpha}$ is the $(1-\alpha)100^{th}$ percentile of a *t* distribution with $(N-1)$ degrees of freedom. The multivariate indexes as well as the confidence interval are presented in Fig. 2. It is evident to conclude that *G* index significantly diverged from confidence interval for mean of *%R&R* between $\mathbf{R_a}$, $\mathbf{R_y}$, $\mathbf{R_z}$, $\mathbf{R_q}$, and $\mathbf{R_t}$. On the other hand, the multivariate indexes proposed in this article were estimated within the confidence interval. In this numerical example, WA_T and WG_T based on $\mathbf{\Sigma_T}$ classify the measurement system as marginal, whereas WA_{MS} and WG_{MS} using $\mathbf{\Sigma_{MS}}$ classify as unacceptable. Further, a more detailed study should be conducted to identify which combination of agglutination and weighting approaches is the most robust in several scenarios involving measurement systems (unacceptable, marginal, and acceptable) and correlation structures between CTQs.

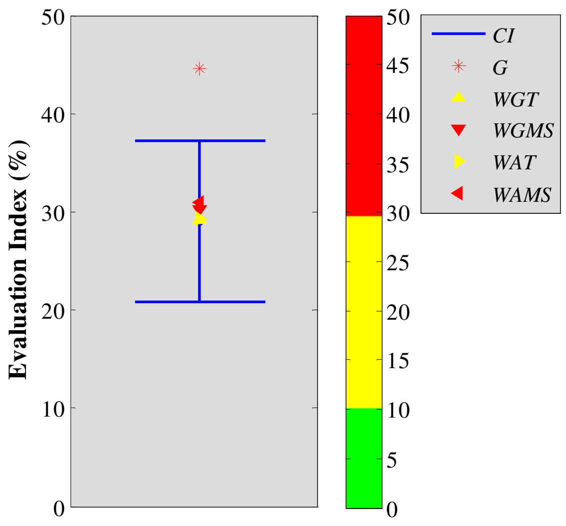


Fig. 2. Confidence interval and multivariate evaluation indexes

5. Conclusions

The present study was designed to address the multivariate analysis of measurement systems through studies of repeatability and reproducibility of the measurement process. Measurements of the roughness parameters, obtained from AISI 12L14 steel turning machined with carbide tools, were evaluated. The weighted indexes were compared to those obtained by the univariate and multivariate indexes in the literature. Statistical analysis provided the following conclusions:

1. The numerical example showed that univariate analysis cannot be satisfactory when correlated characteristics are measured by the same measuring instrument. Multivariate statistical techniques should be used so that a single classification represents the original set of variables.

2. *G* index uses geometric mean for evaluating the measurement system. This approach may be incorrect when the ratio $\sqrt{\lambda_{MS}/\lambda_T}$ for each q pair of eigenvalues provide significant difference for their calculations. Some form of weighting for the calculation of this index must be used;

3. Taking the summarized results in Table 2 into account, this numerical example showed that the additional indexes proposed in this article obtained more coherent results in assessing multivariate GR&R studies. These indexes were able to overcome shortcomings such as: to provide a single assessment for all CTQs in this multivariate GR&R study; and to provide strategies of weighting that guarantee greater importance for the most significant $\sqrt{\lambda_{MS}/\lambda_T}$ ratio.

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