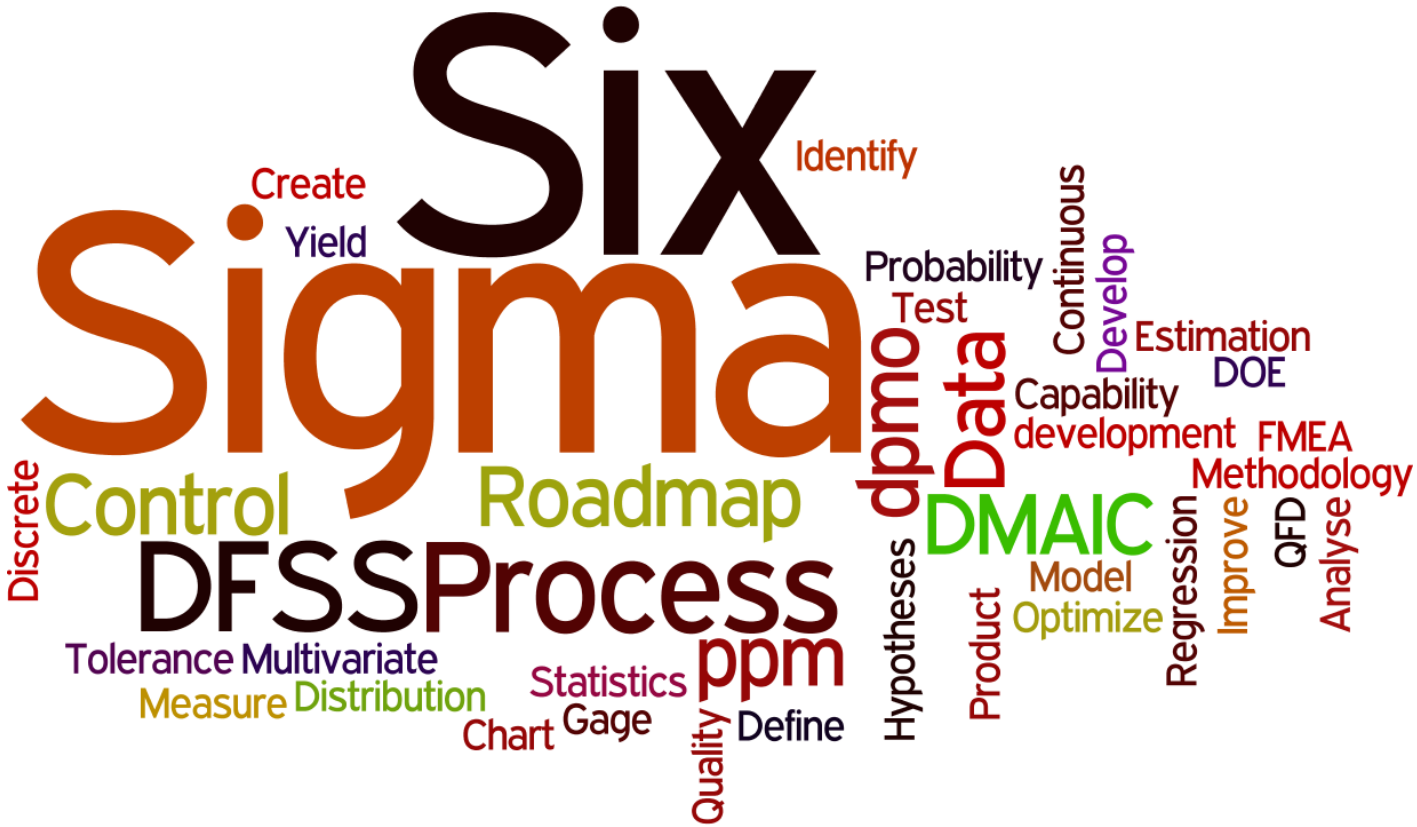


AHP - Examples

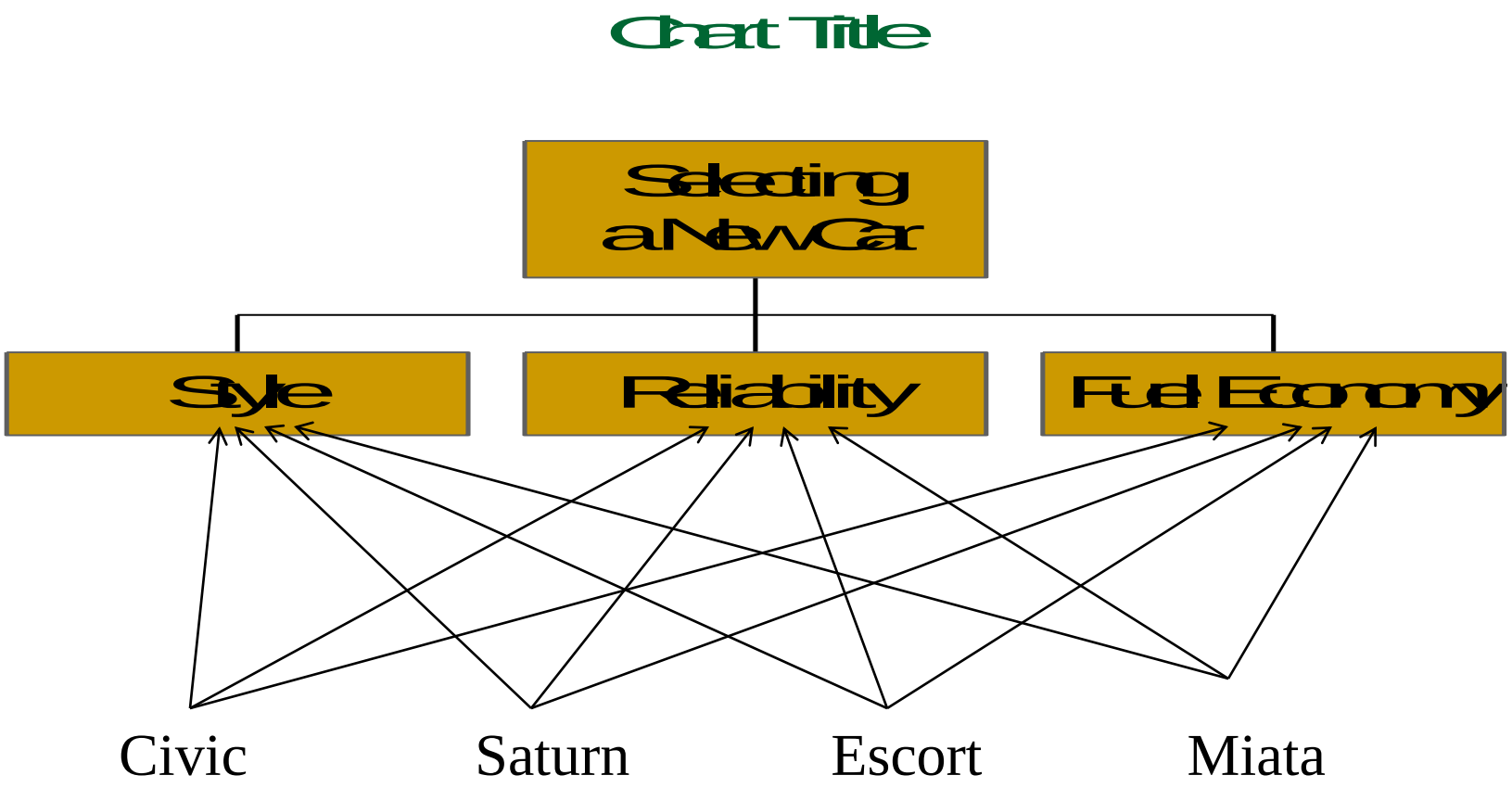
Analytical Hierarchy Process

Green Belts´s Training | Jun 2021

Example: Car Selection

- Objective
  - Selecting a car
- Criteria
  - Style, Reliability, Fuel-economy **Cost?**
- Alternatives
  - Civic Coupe, Saturn Coupe, Ford Escort, Mazda Miata

Hierarchy tree



Alternative courses of action

Ranking of Criteria and Alternatives

- Pairwise comparisons are made with the grades ranging from 1-9.

- A basic, but very reasonable assumption for comparing alternatives:

  If attribute A is absolutely more important than attribute B and is rated at 9, then B must be absolutely less important than A and is graded as 1/9.

- These pairwise comparisons are carried out for all factors to be considered, usually not more than 7, and the matrix is completed.

Ranking Scale for Criteria and Alternatives

Intensity of importance	Definition	Explanation
1	Equal importance	Two factors contribute equally to the objective
3	Somewhat more important	Experience and judgement slightly favour one over the other.
5	Much more important	Experience and judgement strongly favour one over the other.
7	Very much more important	Experience and judgement very strongly favour one over the other. Its importance is demonstrated in practice.
9	Absolutely more important.	The evidence favouring one over the other is of the highest possible validity.
2,4,6,8	Intermediate values	When compromise is needed

Ranking of criteria

Use AHP.xls

	Style	Reliability	Fuel Economy
Style	1	1/2	3
Reliability	2	1	4
Fuel Economy	1/3	1/4	1

Ranking of priorities

- Consider [$Ax = \lambda_{max}x$] where
  - A is the comparison matrix of size n×n, for n criteria, also called the priority matrix.
  - x is the Eigenvector of size n×1, also called the priority vector.
  - λ_{max} is the Eigenvalue, $\lambda_{max} \in \Re > n$.

- To find the ranking of priorities, namely the Eigen Vector X:

  1) Normalize the column entries by dividing each entry by the sum of the column.

  2) Take the overall row averages.

$$A = \begin{bmatrix} 1 & 0.5 & 3 \\ 2 & 1 & 4 \\ 0.33 & 0.25 & 1.0 \end{bmatrix}$$

Column sums 3.33 1.75 8.00

Normalized Column Sums →

$$\begin{bmatrix} 0.30 & 0.29 & 0.38 \\ 0.60 & 0.57 & 0.50 \\ 0.10 & 0.14 & 0.13 \end{bmatrix}$$

1.00 1.00 1.00

Row averages →

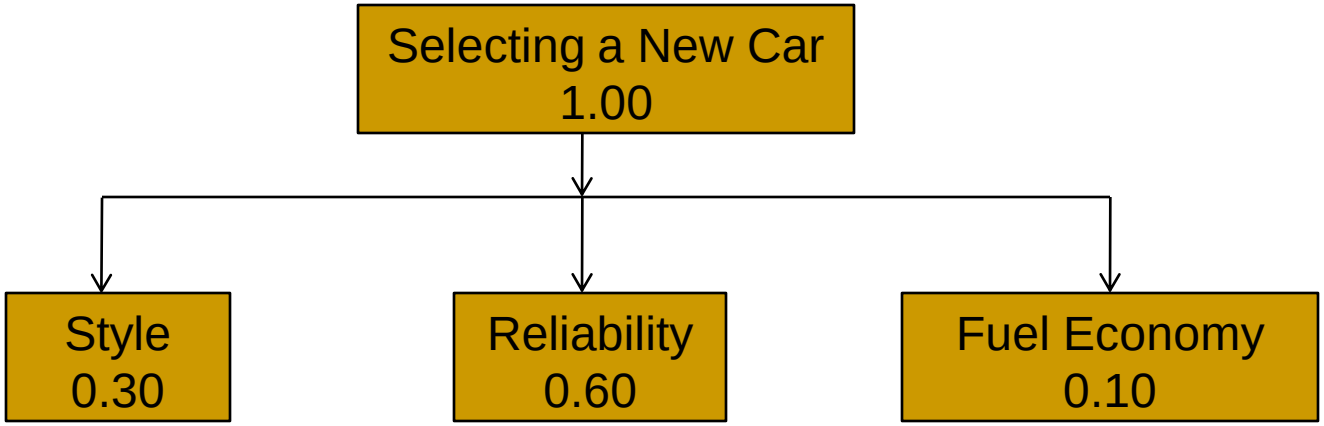
$$X = \begin{bmatrix} 0.30 \\ 0.60 \\ 0.10 \end{bmatrix}$$

Priority vector

Weight in AHP.xls

Criteria weights

- Style .30
- Reliability .60
- Fuel Economy .10



Checking for Consistency

- The next stage is to calculate a Consistency Ratio (CR) to measure how consistent the judgments have been relative to large samples of purely random judgments.

- AHP evaluations are based on the aasumption that the decision maker is rational, i.e., if A is preferred to B and B is preferred to C, then A is preferred to C.

- If the CR is greater than 0.1 the judgments are untrustworthy because they are too close for comfort to randomness and the exercise is valueless or must be repeated.

Calculation of Consistency Ratio

- The next stage is to calculate λ_{max} so as to lead to the Consistency Index and the Consistency Ratio.
- Consider [Ax = λ_{max} x] where x is the Eigenvector.

$$\underset{A}{\begin{bmatrix} 1 & 0.5 & 3 \\ 2 & 1 & 4 \\ 0.333 & 0.25 & 1.0 \end{bmatrix}} \underset{x}{\begin{bmatrix} 0.30 \\ 0.60 \\ 0.10 \end{bmatrix}} = \underset{Ax}{\begin{bmatrix} 0.90 \\ 1.60 \\ 0.35 \end{bmatrix}} = \lambda_{max} \underset{x}{\begin{bmatrix} 0.30 \\ 0.60 \\ 0.10 \end{bmatrix}}$$

λmax=average{0.90/0.30, 1.60/0.6, 0.35/0.10}=3.06

- Consistency index , CI is found by

CI=(λmax-n)/(n-1)=(3.06-3)/(3-1)= 0.03

Consistency Ratio

- The final step is to calculate the Consistency Ratio, CR by using the table below, derived from Saaty's book. The upper row is the order of the random matrix, and the lower row is the corresponding index of consistency for random judgments.

3	4	5	6	7	8	9	10	11	12	13	14	15
0,524	0,881	1,108	1,248	1,342	1,406	1,451	1,495	1,514	1,536	1,555	1,571	1,584

Each of the numbers in this table is the average of CI's derived from a sample of randomly selected reciprocal matrices of AHP method.

An inconsistency of 10% or less implies that the adjustment is small as compared to the actual values of the eigenvector entries.
A CR as high as, say, 90% would mean that the pairwise judgments are just about random and are completely untrustworthy! In this case, comparisons should be repeated.

Consistency in AHP.xls

In the above example: CR=CI/0.58=0.03/0.58=0.05

0.05<0.1, so the evaluations are consistent!

Ranking alternatives

Style	Civic	Saturn	Escort	Miata
Civic	1	1/4	4	1/6
Saturn	4	1	4	1/4
Escort	1/4	1/4	1	1/5
Miata	6	4	5	1

Priority vector

$$\begin{bmatrix} 0.13 \\ 0.24 \\ 0.07 \\ 0.56 \end{bmatrix}$$

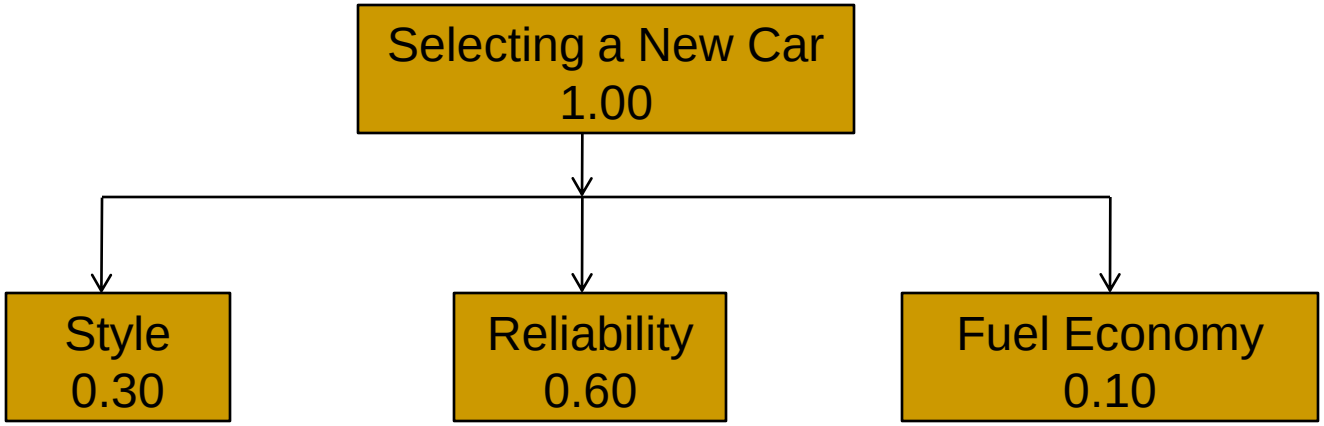
Reliability	Civic	Saturn	Escort	Miata
Civic	1	2	5	1
Saturn	1/2	1	3	2
Escort	1/5	1/3	1	1/4
Miata	1	1/2	4	1

$$\begin{bmatrix} 0.38 \\ 0.29 \\ 0.07 \\ 0.26 \end{bmatrix}$$

Ranking alternatives

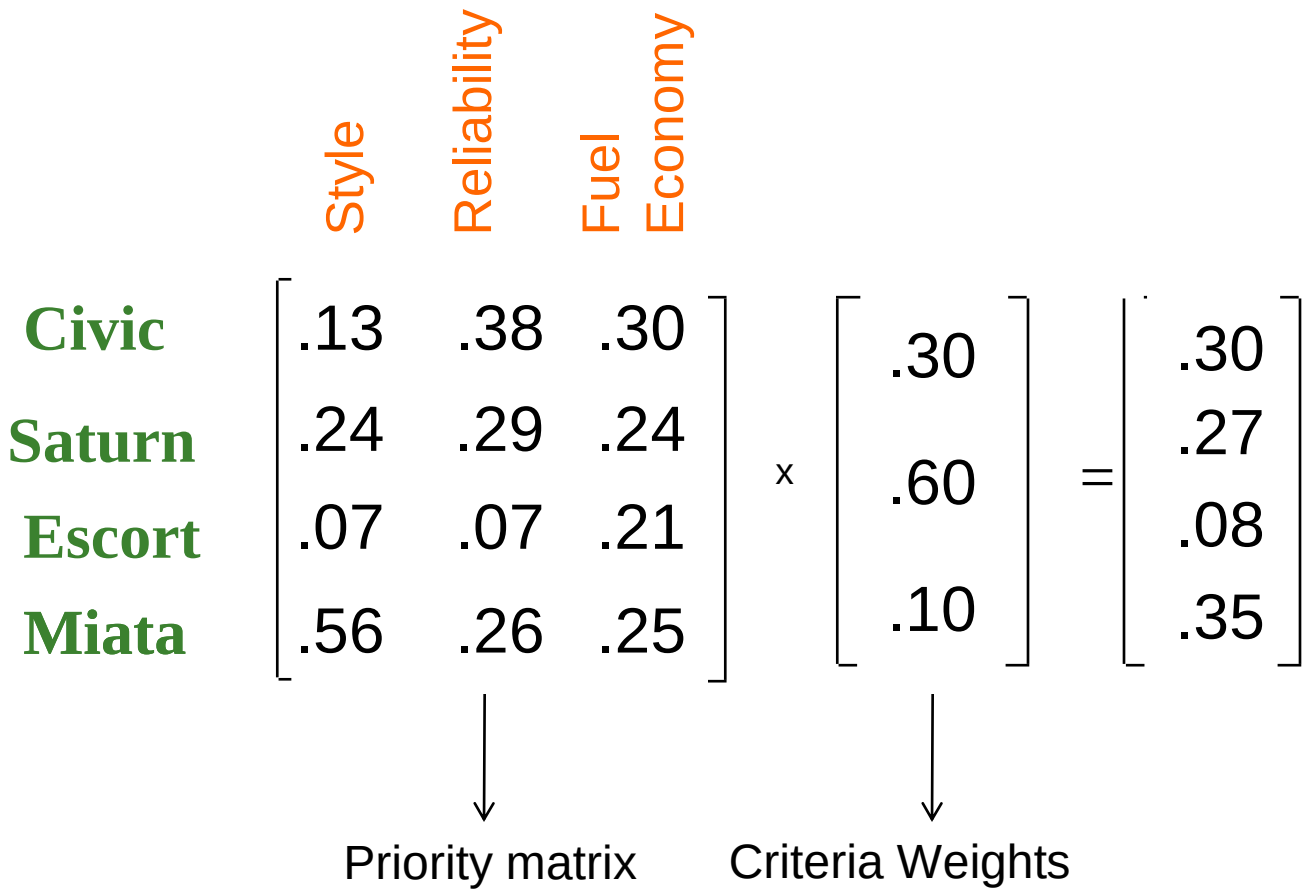
		Miles/gallon	Normalized
Fuel Economy	Civic	34	.30
	Saturn	27	.24
	Escort	24	.21
	Miata	28	.25
		113	1.0

! Since fuel economy is a quantitative measure, fuel consumption ratios can be used to determine the relative ranking of alternatives; however this is not obligatory. Pairwise comparisons may still be used in some cases.



Style		Reliability		Fuel Economy	
Civic	0.13	Civic	0.38	Civic	0.30
Saturn	0.24	Saturn	0.29	Saturn	0.24
Escort	0.07	Escort	0.07	Escort	0.21
Miata	0.56	Miata	0.26	Miata	0.25

Ranking of alternatives



Including **Cost** as a Decision Criteria

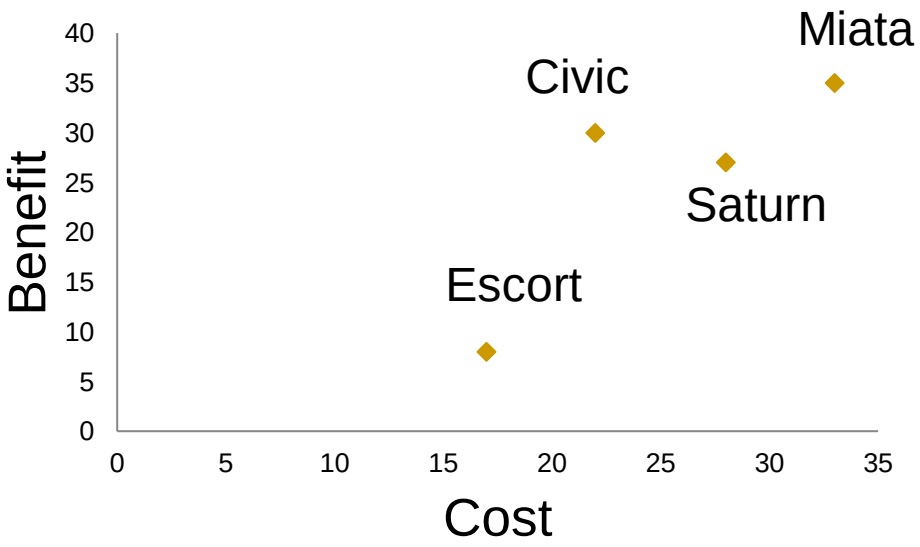
Adding “cost” as a a new criterion is very difficult in AHP. A new column and a new row will be added in the evaluation matrix. However, whole evaluation should be repeated since addition of a new criterion might affect the relative importance of other criteria as well!

Instead one may think of normalizing the costs directly and calculate the cost/benefit ratio for comparing alternatives!

	Cost	Normalized Cost	Benefits	Cost/Benefits Ratio
CIVIC	$12K	.22	.30	0.73
SATURN	$15K	.28	.27	1.03
ESCORT	$9K	.17	.08	2.13
MIATA	$18K	.33	.35	0.92

Methods for including cost criterion

- Use graphical representations to make trade-offs.



- Calculate cost/benefit ratios
- Use linear programming
- Use seperate benefit and cost trees and then combine the results

Ex 2: Evaluation of Job Offers

Ex: Peter is offered 4 jobs from Acme Manufacturing (A), Bankers Bank (B), Creative Consulting (C), and Dynamic Decision Making (D).
He bases his evaluation on the criteria such as location, salary, job content, and long-term prospects.

Step 1: Decide upon the relative importance of the selection criteria:

	Location	Salary	Content	Long-term
Location	**1**	**1/5**	**1/3**	**1/2**
Salary	**5**	**1**	**2**	**4**
Content	**3**	**1/2**	**1**	**3**
Long-term	**2**	**1/2**	**1/3**	**1**

Priority Vectors:

1) Normalize the column entries by dividing each entry by the sum of the column.
2) Take the overall row averages

	Location	Salary	Content	Long-term	Average
Location	0.091	0.102	0.091	0.059	0.086
Salary	0.455	0.513	0.545	0.471	0.496
Content	0.273	0.256	0.273	0.353	0.289
Long-term	0.182	0.128	0.091	0.118	0.130
	+				+
	1	1	1	1	1

Example 2: Evaluation of Job Offers

Step 2: Evaluate alternatives w.r.t. each criteria

Location Scores

	A	B	C	D
A	1	1/2	1/3	5
B	2	1	1/2	7
C	3	2	1	9
D	1/5	1/7	1/9	1

Relative Location Scores

	A	B	C	D	Avg.
A	0.161	0.137	0.171	0.227	0.174
B	0.322	0.275	0.257	0.312	
C	0.484	0.549	0.514	0.409	0.293
D	0.032	0.040	0.057	0.045	
					0.489
					0.044

Example 2: Calculation of Relative Scores

	Relative Scores for Each Criteria					Relative weights for each criteria		Relative scores for each alternative
	Location	**Salary**	**Content**	**Long-Term**				
A	0.174	0.050	0.210	0.510	x	0.086	=	0.164
B	0.293	0.444	0.038	0.012		0.496		0.256
C	0.489	0.312	0.354	0.290		0.289		0.335
D	0.044	0.194	0.398	0.188		0.130		0.238

Example 3: AHP in project management

Step 1: Evaluation of the weights of the criteria

Pair-wise comparison matrix for the six criteria[a]

	Exp.	FS	QP	MPR	ER	CWL	Priority vector
Exp.	1	2	3	6	6	5	0.372
FS	1/2	1	3	6	6	5	0.293
QP	1/3	1/3	1	4	4	3	0.156
MPR	1/6	1/6	1/4	1	2	1/2	0.053
ER	1/6	1/6	1/4	1/2	1	1/4	0.039
CWL	1/5	1/5	1/3	2	4	1	0.087
							$\sum = 1.00$

[a] $\lambda_{max} = 6.31$, CI = 0.062, RI = 1.24, CR = 0.05 < 0.1 OK.

Step 2: a) Pairwise comparison matrix for experience

Exp.	A	B	C	D	E
A	1	1/3	1/2	1/6	2
B	3	1	2	1/2	4
C	2	1/2	1	1/3	3
D	6	2	3	1	7
E	1/2	1/4	1/3	1/7	1

Exp.	A	B	C	D	E	Priority vector
A	0.08	0.082	0.073	0.078	0.118	0.086
B	0.24	0.245	0.293	0.233	0.235	0.249
C	0.16	0.122	0.146	0.155	0.176	0.152
D	0.48	0.489	0.439	0.466	0.412	0.457
E	0.04	0.061	0.049	0.066	0.059	0.055
						$\sum = 0.999$

[a] $\lambda_{max} = 5.037$, CI = 0.00925, RI = 1.12, CR = 0.0082 < 0.1 OK.

Example 3: AHP in project management

Calculation of priority vector:

	Exp. (0.372)	FS (02.93)	QP (0.156)	MPR (0.053)	ER (0.039)	CWL (0.087)
A	0.086	0.425	0.269	0.151	0.084	0.144
B	0.249	0.088	0.074	0.273	0.264	0.537
C	0.152	0.178	0.461	0.449	0.556	0.173
D	0.457	0.268	0.163	0.081	0.057	0.084
E	0.055	0.039	0.031	0.045	0.038	0.062

$$\times \begin{bmatrix} 0.372 \\ 0.293 \\ 0.156 \\ 0.053 \\ 0.039 \\ 0.087 \end{bmatrix} = \begin{bmatrix} 0.222 \\ 0.201 \\ 0.241 \\ 0.288 \\ 0.046 \end{bmatrix}$$

Probably Contractor-E should have been eliminated. It appears to be the worst.

Note that a DSS supports the decision maker, it can not replace him/her. Thus, an AHP Based DSS should allow the decision maker to make sensitivity analysis of his judgements on the overall priorities !

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